Math Analysis

The Following is the list of all kind of Math Analysis courses that OMATHA can offer either online or home tutoring

Elementary Real Analysis

Review of the completeness properties of real numbers. Supremum and infimum, lim sup, lim inf. The topology of Rén. Uniform continuity. Compactness, Heine-Borel. The Riemann integral, the fundamental theorem of calculus, improper integrals. Sequences and series of functions, uniform convergence, and Fourier series. Note: This course is mostly intended for those students whose majors might be Mathematics or Statistics.

Elementary or Introductory Mathematical Analysis This course is for students who have successfully completed Calculus I & II. This course presents foundation concepts in analysis. It is normally required material for mathematics majors. Topics studied include the nature of proof, set theory and cardinality, the real numbers, limits of sequences and functions, continuity, formal coverage of the derivative and the mean value theorem, Taylor's theorem, the Riemann integral, the fundamental theorem of calculus, and topics in infinite series.

Elements of Real Analysis

Metric spaces, continuous functions. Compactness and connectedness. Contraction mappings. The inverse function theorem and the implicit function theorem. Series of functions; modes of convergence, power series, Fourier series. Topics on function spaces such as: Weierstrass approximation, L^2 spaces.

Note: This course is intended for Pure and Applied Mathematics Students.

Fundamental Concepts of Analysis

Recommended for Mathematics majors and required of honors Mathematics majors. A more advanced and general version of Math 115, introducing and using metric spaces. Properties of Riemann integrals, continuous functions and convergence in metric spaces; compact metric spaces, basic point set topology.

Lebesgue Integration and Fourier Analysis Similar to 205A, but for undergraduate Math majors and graduate students in other disciplines. Topics include Lebesgue measure on Euclidean space, Lebesgue integration, L^p spaces, the Fourier transform, the Hardy-Littlewood maximal function and Lebesgue differentiation.

Mathematical Analysis I

This Course is intended to serve as a first course in analysis that is usually taken by advanced undergraduates or by first-year students who study mathematics. The contents of this course are the fo;;oeing:

- The Real and Complex Number Systems (Introduction, Ordered Sets, Fields, The Real Field, The ٠ Extended Real Number System, The Complex Field, Euclidean Spaces)
- **Basic Topology** (Finite, Countable, and Uncountable Sets, Metric Spaces, Compact Sets, Perfect Sets, Connected Sets)
- Numerical Sequences and Series (Convergent Sequences, Subsequences, Cauchy Sequences, Upper and Lower Limits, Some Special Sequences, Series, Series of Nonnegative Terms, The Number e, The Root and Ratio Tests, Power Series, Summation by Parts, Absolute Convergence, Addition and Multiplication of Series, Rearrangements)
- **Continuity** (Limits of Functions, Continuous Furlctions, Continuity and Compactness, Continuity and Connectedness, Discontinuities, Monotonic Functions, Infinite Limits and Limits at Infinity)

Mathematical Analysis II

This course is intended to serve as a second course in analysis that is usually taken by advanced first-year undergraduate students who study mathematics. The contents of this course are the following:

- **Differentiation** (The Derivative of a Real Function, Mean Value Theorems, The Continuity of Derivatives, L'Hospital's Rule, Derivatives of Higher Order, Taylor's Theorem, Differentiation of Vector-valued Functions)
- **The Riemann-Stieltjes Integral** (Definition and Existence of the Integral, Properties of the Integral, Integration and Differentiation, Integration of Vector-valued Functions, Rectifiable Curves)
- Sequences and Series of Functions (Discussion of Main Problem, Uniform Convergence Uniform Convergence and Continuity, Uniform Convergence and Integration, Uniform Convergence and Differentiation, Equicontinuous Families of Functions, The Stone-Weierstrass Theorem)
- Some Special Functions (Power Series, The Exponential and Logarithmic Functions, The Trigonometric Functions, The Algebraic Completeness of the Complex Field, Fourier Series, The Gamma Function)

Mathematical Analysis III

This course is intended to serve as a third course in analysis that is usually taken by senior undegraduate or first-year graduate students who study mathematics. The contents of this course are the following:

- Functions of Several Variables (Linear Transformations, Differentiation, The Contraction Principle, The Inverse Function Theorem, The Implicit Function Theorem, The Rank Theorem, Determinants, Derivatives of Higher Order, Differentiation of Integrals)
- Integration of Differential Forms (Integration, Primitive Mappings, Partitions of Unity, Change of Variables, Differential Forms, Simplexes and Chains, Stokes' Theorem, Closed Forms and Exact Forms, Vector Analysis)
- The Lebesgue Theory (Set Functions, Construction of the Lebesgue Measure, Measure Spaces, Measurable Functions, Simple Functions, Integration, Comparison with the Riemann Integral, Integration of Complex Functions, Functions of Class L^2)

Vector Analysis

This course is an introduction to vector analysis, and is an honors version of Calculus of Several Variables.. The material covered will be a strict super-set of 268, and more emphasis will be placed on writing rigorous proofs. The treatment of differential calculus will be through and rigorous. In the interest of time, however, many results on integral calculus will be stated without proof, or proved under simplifying assumptions.

A Tentative Syllabus for this course is: Functions of several variables, regions and domains, limits and continuity. Sequential compactness. Partial derivatives, linearization, Jacobian. Chain rule, inverse and implicit functions and geometric applications. Higher derivatives, Taylor's theorem, optimization, vector fields. Multiple integrals and change of variables, Leibnitz's rule. Line integrals, Green's theorem. Path independence and connectedness, conservative vector fields. Surfaces and orientability, surface integrals. Divergence theorem and Stokes's theorem.

Elementary Complex Analysis

This is an introductory course to Complex Analysis at an undergraduate level. Complex Analysis, in a nutshell, is the theory of differentiation and integration of functions with complex-valued arguments. While the course will try to include rigorous proofs for many - but not all - of the material covered, emphasize will be placed on applications and examples. Complex Analysis is a topic that is extremely useful in many applied topics such as numerical analysis, electrical engineering, physics, chaos theory, and much more, and you will see some of these applications throughout the course. In addition, complex analysis is a subject that is, in a sense, very complete. The concept of complex differentiation is much more restrictive than that of real differentiation and as a result the corresponding theory of complex differentiable functions is a particularly nice one - as you will hopefully agree at the end of the course. The course will cover the following materials that are considered standard for an undergraduate complex analysis course:

1. Complex Numbers (Basic Algebraic, Vectors and Moduli, Conjugates, Exponentials, Products and Powers, Roots, Regions in the Complex Plane)

- 2. Analytic Functions (Limits, Continuity, Derivatives, Cauchy Riemann Equations, Analytic Functions, Harmonic Functions)
- 3. Elementary Functions (Exponential, Logarithm, Complex Exponents, Trigs, Hyperbolic Functions)
- 4. Integrals (Definite Integrals, Contour Integrals, Antiderivatives, Cauchy Goursat Theorem, Cauchy Integral Formula, Liouville's Theorem, Fundamental Theorem of Algebra, Maximum Modulus Principle)
- 5. Series (Sequences, Convergence of Series, Taylor Series, Laurent Series, Absolute and Uniform Convergence, Power Series techniques)
- 6. Residues and Poles (Residues, Cauchy's Residue Theorem, Residue at Infinity, Zeros of Analytic Functions)

We might also cover excerpts from "Applications of Residues), "Mapping by Elementary Functions", or some "Dynamic Systems", depending on how the course progresses.

Elementary Functional Analysis

Introduction, inner product spaces, Normed spaces, Hilbert and Banach spaces, Completions, Orthogonal expansions, Classical Fourier series, Dual spaces, Linear operators (Ch 7) February 3. Dual spaces (Ch 6) February 8. Linear operators, Compact operators, Sturm-Liouville systems, Green's functions, Eigenfunction expansions.

Real Analysis I

Calculus of one and several variables, the Implicit and Inverse Function Theorems, pointwise and uniform convergence of sequences of functions, integration and differentiation of sequences, the Weierstrass Approximation Theorem, Lebesgue measure and integration on the real line, Measurable sets, Lebesgue measure, measurable functions, the Lebesgue integral and its relation to the Riemann integral, convergence theorems, functions of bounded variation, absolute continuity and differentiation of integrals, General measure and integration theory, Measure spaces, measurable functions, integration convergence theorems, signed measures, the Radon-Nikodym Theorem, product measures, Fubini's Theorem, Tonelli's Theorem. operators; applications to integral equations.

Real Analysis II

Families of functions, Equicontinuous families and the Arzela-Ascoli Theorem, the Stone-Weierstrass Theorem. Banach spaces, L^P- spaces and their conjugates, the Riesz-Fisher Theorem, the Riesz Representation Theorem for bounded linear functionals on L^P, C(X), the Riesz Representation Theorem for C(X), the Hahn-Banach Theorem, the Closed Graph and Open Mapping Theorems, the Principle of Uniform Boundedness, Alaoglu's Theorem, Hilbert spaces, orthogonal systems, Fourier series, Bessel's inequality, Parseval's formula, convolutions, Fourier transform, distributions, Sobolev spaces, and Radon measures.

Advanced Complex Analysis

This is the first part of a series of lectures on advanced topics in Complex Analysis. By advanced, we mean topics that are not (or just barely) touched upon in a first course on Complex Analysis. The theme of the course is to study zeros of analytic (or holomorphic) functions and related theorems. These include the theorems of Hurwitz and Rouche, the Open Mapping theorem, the Inverse and Implicit Function theorems, applications of those theorems, behaviour at a critical point, analytic branches, constructing Riemann surfaces for functional inverses, Analytic continuation and Monodromy, Hyperbolic geometry and the Riemann Mapping theorem.

Introduction to Numerical Analysis

Numerical analysis is a discipline of mathematics concerned with the development of efficient methods for getting numerical solutions to complex mathematical problems. There are three sections to the numerical analysis. The first section of the subject deals with the creation of a problem-solving approach. The analysis of methods, which includes error analysis and efficiency analysis, is covered in the second section. The efficiency analysis **shows us** how fast we can compute the result, while the error analysis informs us how correct the result will be if we utilize the approach. The construction of an efficient algorithm to implement the approach as a computer code is the

3

subject's third part. All three elements must be familiar to have a thorough understanding of the numerical analysis. Topics spanned root finding, interpolation, approximation of functions, integration, differential equations and direct and iterative methods in linear algebra.

Applied Analysis

Interior Point Methods: exposes students to the modern IPM theory with some applications, to the extent that at the end of the course a student should be able to implement a basic IPM algorithm.

Theoretical Numerical Analysis: provides the theoretical underpinnings for the analysis of modern numerical methods, covering topics such as linear operators on normed spaces, approximation theory, nonlinear equations in Banach spaces, Fourier analysis, Sobolev spaces and weak formulations of elliptic boundary value problems, with applications to finite difference, finite element and wavelet methods.

Differential Equations: essential ideas relating to the analysis of differential equations from a functional analysis point of view. General topics include Hilbert spaces and the Lax-Milgram's theorem, variational formulation of boundary value problems, finite element methods, Sobolev spaces, distributions, and pseudo-differential operators.

Functional Analysis

This course is a graduate level course in functional analysis. Classically, functional analysis is the study of infinite dimensional vector spaces of functions and linear operators between them. This class deals with relevant function spaces (normed vector spaces, Banach and Hilbert spaces), bounded linear operators on normed vector spaces, fundamental principles of functional analysis (i.e., Han-Banach Theorem, Uniform Boundedness Principle, Open Mapping Theorem and Closed Graph Theorem) and their applications, spectral theory of compact linear operators and spectral theory of compact self-adjoint operators. The goal of the course is to help students who pursue advanced studies in mathematics and related fields to lay a solid foundation in functional analysis. We will cover chapters 1-9 in the textbook.

Fourier Analysis

Half of the subject of this is devoted to the theory of the Lebesgue integral with applications to probability, and the other half to Fourier series and Fourier integrals. The task in the first half of the course is to introduce Lebesgue measure and establish properties of the Lebesgue integral. Our textbook (Adams and Guillemin) introduces Lebesgue measure using motivation and examples from probability theory. After we have developed probability theory on Bernoulli sequences, using a corresponding with Lebesgue measure on the unit interval, we will discuss the Lebesgue integral and some Fourier analysis. Then we will use some Fourier analysis to prove more theorems in probability. By the end of the semester we will have all the tools to discuss the continuum limit of a (suitably scaled) random walk, namely Brownian motion.

One of the main goals this course is to establish rules for the limiting behavior of functions so that we can deal with functions with as much confidence as we do real or complex numbers. An equally important motivation (that will only become clear in the second half) is that the systematic study of Fourier series requires the Lebesgue integral. The square mean convergence of Fourier series and Parseval's formula cannot be stated accurately in proper generality without the Lebesgue integral and Lebesgue integrable functions.

Elementary Harmonic Analysis

In the first part of the course, we will study basic concepts in Harmonic Analysis, specifically, interpolation theorems, Hardy-Littlewood maximal function and the Hardy-Littlewood-Sobolev fractional integration theorem. This will be followed by the Littlewood-Paley theory and their applications in r-variational estimates as well as the Calderon-Zygmund theory of singular integrals and Radon operators. In the second part of the course we will study restriction theorems and Kakeya maximal functions, we will also construct the Besicovitch set. This will be a starting point of the decoupling theory, which will be used to prove the Vinogadov mean value conjecture from number theory. Finally, we will prove the Carleson theorem, which asserts that partial sums of Fourier series of a square integrable function converge pointwise almost everywhere.