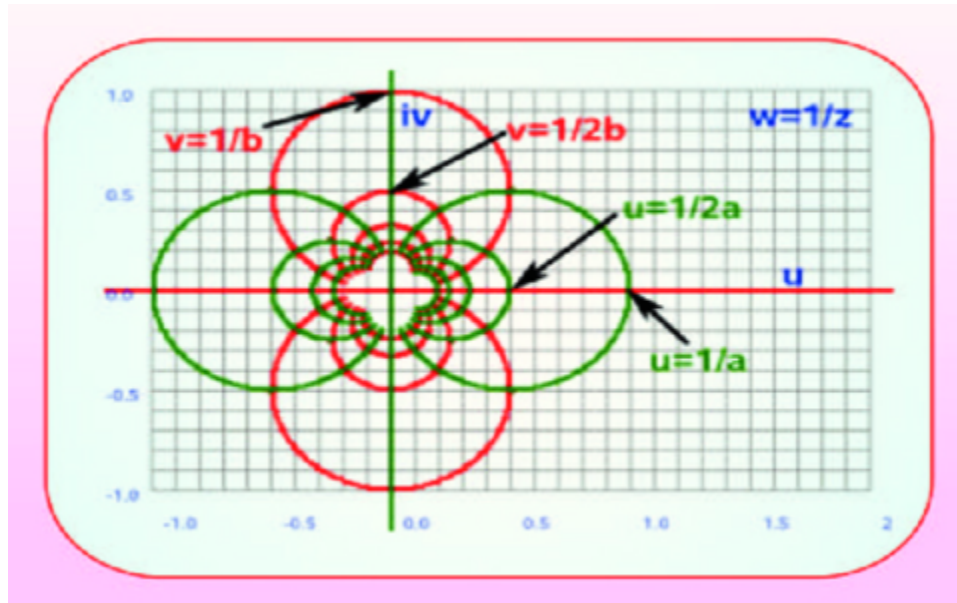


# Applied Complex Variables for Scientists & Engineers

This is an introduction to complex variable methods for scientists and engineers. It begins by carefully defining complex numbers and analytic functions, and proceeds to give accounts of complex integration, Taylor series, singularities, residues and mappings. Both algebraic and geometric tools are employed to provide the greatest understanding, with many diagrams illustrating the concepts introduced. The emphasis is laid on understanding the use of methods, rather than on rigorous proofs. One feature that will appeal to scientists is the high proportion of the book devoted to applications of the material to physical problems. These include detailed treatments of potential theory, hydrodynamics, electrostatics, gravitation and the uses of the Laplace transform for partial differential equations. The text contains some 300 stimulating exercises of high quality, with solutions given to many of them. It will be highly suitable for students wishing to learn the elements of complex analysis in an applied context.



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