Introduction to Complex Variables

Description

The study of complex variables is beautiful from a purely mathematical point of view, and very useful for solving a wide array of problems arising in applications. This introduction to complex variables, suitable as a text for a one-semester course, has been written for undergraduate students in applied mathematics, science, and engineering. Based on the authors' extensive teaching experience, it covers topics of keen interest to these students, including ordinary differential equations, as well as Fourier and Laplace transform methods for solving partial differential equations arising in physical applications. Many worked examples, applications, and exercises are included. With this foundation, students can progress beyond the standard course and explore a range of additional topics, including generalized Cauchy theorem, Painlevé equations, computational methods, and conformal mapping with circular arcs. Advanced topics can be included in the syllabus or form the basis for challenging student projects.





Syllabuses is in next pages

CONTENTS

1	Complex Numbers	1
	Sums and Products 1	
	Basic Algebraic Properties 3	
	Further Properties 5	
	Moduli 8	
	Complex Conjugates 11	
	Exponential Form 15	
	Products and Quotients in Exponential Form 17 Roots of Complex Numbers 22	
	Examples 25	
	Regions in the Complex Plane 29	
2	Analytic Functions	33
	Functions of a Complex Variable 33	
	Mappings 36	
	Mappings by the Exponential Function 40	
	Limits 43	
	Theorems on Limits 46	
	Limits Involving the Point at Infinity 48	
	Continuity 51	
	Derivatives 54	
	Differentiation Formulas 57	
	Cauchy–Riemann Equations 60	

Preface

xv

Sufficient Conditions for Differentiability 63 Polar Coordinates 65 Analytic Functions 70 Examples 72 75 Harmonic Functions Uniquely Determined Analytic Functions 80 Reflection Principle 82 3 Elementary Functions 87 The Exponential Function 87 90 The Logarithmic Function Branches and Derivatives of Logarithms 92 Some Identities Involving Logarithms 95 Complex Exponents 97 100 Trigonometric Functions 105 Hyperbolic Functions Inverse Trigonometric and Hyperbolic Functions 108 111 4 Integrals Derivatives of Functions w(t) 111 Definite Integrals of Functions w(t)113 Contours 116 122 Contour Integrals 124 Examples Upper Bounds for Moduli of Contour Integrals 130 Antiderivatives 135 138 Examples Cauchy-Goursat Theorem 142 Proof of the Theorem 144 Simply and Multiply Connected Domains 149 Cauchy Integral Formula 157 Derivatives of Analytic Functions 158 Liouville's Theorem and the Fundamental Theorem of Algebra 165 Maximum Modulus Principle 167 5 175 Series Convergence of Sequences 175 Convergence of Series 178 Taylor Series 182 185 Examples 190 Laurent Series 195 Examples Absolute and Uniform Convergence of Power Series 200 Continuity of Sums of Power Series 204 Integration and Differentiation of Power Series 206 Uniqueness of Series Representations 210 Multiplication and Division of Power Series 215

6	Residues and Poles	221
	Residues 221	
	Cauchy's Residue Theorem 225	
	Using a Single Residue 227	
	The Three Types of Isolated Singular Points 231	
	Residues at Poles 234	
	Zeros of Analytic Eurotions 230	
	Zeros and Poles 242	
	Behavior of f Near Isolated Singular Points 247	
	Demarter ery riem technice emganar reinte	
-		
7	Applications of Residues	251
	Evaluation of Improper Integrals 251	
	Example 254	
	Improper Integrals from Fourier Analysis 259	
	Jordan's Lemma 262	
	An Indentation Around a Branch Point 270	
	Integration Along a Branch Cut 273	
	Definite Integrals involving Sines and Cosines 278	
	Argument Principle 281	
	Rouché's Theorem 284	
	Inverse Laplace Transforms 288	
	Examples 291	
8	Mapping by Elementary Functions	299
Ŭ	Linear Transformations 200	2//
	The Transformation $w = 1/z$ 301	
	Mappings by $1/z$ 303	
	Linear Fractional Transformations 307	
	An Implicit Form 310	
	Mappings of the Upper Half Plane 313	
	The Transformation $w = \sin z$ 318	
	Mappings by z^2 and Branches of $z^{1/2}$ 324	
	Square Roots of Polynomials 329	
	Riemann Surfaces 335	
	Surfaces for Related Functions 3.38	
9	Conformal Mapping	343
	Preservation of Angles 343	
	Scale Factors 346	
	Local Inverses 348	
	Harmonic Conjugates 351	
	Transformations of Harmonic Functions 353	
	Transformations of Boundary Conditions 355	

Applications of Conformal Mapping	361
Steady Temperatures 361	
Steady Temperatures in a Half Plane 363	
A Related Problem 365	
Temperatures in a Quadrant 368	
Electrostatic Potential 373	
Potential in a Cylindrical Space 374	
Two-Dimensional Fluid Flow 379	
The Stream Function 381	
Flows Around a Corner and Around a Cylinder 383	
The Schwarz-Christoffel Transformation	391
Mapping the Real Axis onto a Polygon 391	
Schwarz-Christoffel Transformation 393	
Triangles and Rectangles 397	
Degenerate Polygons 401	
Fluid Flow in a Channel Through a Slit 406	
Flow in a Channel with an Offset 408	
Electrostatic Potential about an Edge of a Conducting Plate 411	
Integral Formulas of the Poisson Type	417
Poisson Integral Formula 417	
Dirichlet Problem for a Disk 419	
Related Boundary Value Problems 423	
Schwarz Integral Formula 427	
Dirichlet Problem for a Half Plane 429	
Neumann Problems 433	
Appendixes	437
Bibliography 437	
Table of Transformations of Regions 441	
Index	451
	Applications of Conformal MappingSteady Temperatures 361Steady Temperatures in a Half Plane 363A Related Problem 365Temperatures in a Quadrant 368Electrostatic Potential 373Potential in a Cylindrical Space 374Two-Dimensional Fluid Flow 379The Stream Function 381Flows Around a Corner and Around a Cylinder 383The Schwarz-Christoffel TransformationMapping the Real Axis onto a Polygon 391Schwarz-Christoffel Transformation 393Triangles and Rectangles 397Degenerate Polygons 401Fluid Flow in a Channel Through a Slit 406Flow in a Channel Through a Slit 406Flow in a Channel Through a Slit 408Electrostatic Potential about an Edge of a Conducting Plate 411Integral Formulas of the Poisson TypePoisson Integral Formula 417Dirichlet Problem for a Disk 419Related Boundary Value Problems 423Schwarz Integral Formula 427Dirichlet Problem for a Half Plane 429Neumann Problems 433AppendixesBibliography 437Table of Transformations of Regions 441Index