Special Courses In Mathematics

The Following is the list of Special Courses in Mathematics that OMATHA can offer either online or in-person

Mathematical Methods I

Review of elementary functions. Limits. Geometric series. Differential and integral calculus in one variable with applications. Functions of several variables. Partial derivatives.

Note: This course is intended primarily for students in the School of Management.

Mathematical Methods II

Solution of systems of linear equations. Matrix algebra. Determinants. Complex numbers, fundamental theorem of algebra. Eigenvalues and eigenvectors of real matrices. Introduction to vector spaces, linear independence, bases. Applications. This course is intended primarily for students in the School of Management and the Faculty of Social Sciences.

Note: This course is the continuation of Mathematical Methods II and is intended primarily for students in the School of Management.

Mathematical Methods III

Sequences, series, power series, Taylor series. Difference equations: the general solution of linear equations with constant coefficients. Additional techniques of integration. Improper integrals. Chain rule for functions of several variables. Gradient, Directional derivative, tangent plane. Partial derivatives of higher order. Extreme with or without constraints.

Note: This course is the continuation of Mathematical Methods III and is intended primarily for students in the School of Management.

Intensive Mathematical Methods I

Instantaneous rate of change and definition of limits; continuity. Derivatives of polynomials using limits, derivatives of sums, products, the chain rule, derivatives of rational, exponential, and radical functions. Derivatives of quotients, logarithms. Analysis of functions via the first and the second derivatives. Applications to finding maxima and minima. Concavity and points of inflection, and graph sketching; Implicit differentiation, related rates, optimization. Geometric series. Integral calculus in one variable with applications. Functions of several variables. Partial derivatives.

Note: This course is intended for those whose are interested in applied mathematics or have chosen applied mathematics as their major.

Mathematical Reasoning and Proofs

Elements of logic, set theory, functions, equivalence relations and cardinality. Proof techniques. Concepts are introduced using sets of integers, integers modulo n, rational, real and complex numbers. Exploration of the real line: completeness, supremum, sequences and limits. Some of the concepts will be illustrated with examples from geometry, algebra and number theory.

Note: This course is intended for students whose majors will be Mathematics or Pure mathematics and those students who are interested in the application of mathematical reasoning.

Foundations of Mathematics
Introduction to proofs, set theory and the foundations of mathematics. Propositional logic, introduction to predicate logic and axiomatic theories. Proof techniques (direct, by contradiction, by cases, constructive and non constructive, induction). Informal set theory (sets, functions, equivalence relations, order relations). Paradoxes. Introduction to axiomatic set theory and to the encoding of mathematics. Axiom of Choice, Zorn's Lemma. Cardinality of sets.

Note: This course is intended for Pure Mathematics students and those who are interested ln mathematical logic.

Foundations of Probability
An overview of probability from a non-measure theoretic point of view, random vectors, independence, conditional expectation and probability, consequences. various types of convergence leading to proofs of the major theorems in classical probability theory, an introduction to simple stochastic processes such as Poisson and branching processes.

Note: This course is intended for Pure Mathematics and Statistics students. Applied Mathematics and Engineering students are alsp welcome.

Mathematics for Engineers
Series solutions of ordinary differential equations, Legendre and Bessel functions, Sturm-Liouville problems, orthogonal functions, Fourier series, Partial differential equations, introduction and applications.

Note: This course is intended only for Engineering students.

Financial Mathematics
Review of conditional expectation and an introduction to martingales, stopping times and the Snell envelope, Interest rate and present value, Discrete time option pricing, Review of the multivariate normal with applications to Markovitz portfolio theory, An introduction to Brownian motion and the Black-Scholes formula for European options.

Note: This course is intended for Statistics and Applied Mathematics Students. Pure Mathematics students are also welcome.

Introduction to Numerical Methods
Roots of nonlinear equations (fixed point, Newton, secant, bisection). Condition number of linear systems. Iterative methods for linear and non-linear systems (Gauss-Seidel, Gauss-Jacobi, SOR; fixed point, Newton) . Interpolation and polynomial approximation, numerical differentiation and integration. Numerical methods for differential equations. Error analysis.

Note: This course is intended for Mathematics and Statistics students. Engineering students are also welcome.

Introduction to Mathematical Models
Introduction to modelling and to mathematical techniques used in applications, Mathematical models will come from various areas of applied sciences and use techniques from calculus, Differential equations, Linear algebra and vector geometry.

Note: This course is intended for all those students who needs to use their knowledge in mathematics to describe a real world problem in mathematical terms, usually in the form of equations.

Measure and Integration I
General measure and integral, Lebesgue measure and integration on R, Fubini's theorem, Lebesgue-Radon-Nikodymtheorem, Absolute continuity and differentiation, Lp-spaces.

Note: This course is intended for all graduate Mathematics or Statistics students. All other interested students who may understand the contents of this course and are at the above indicated level of studies are also welcome.

Measure and Integration II

Banach and Hilbert spaces, bounded linear operators, dual spaces, some additional topics.

Note: This course is intended for all graduate Pure and Applied Mathematics students.

Special Topics in Mathematics

Selected advanced topics in Fourier Analysis, Harmonic Analysis, Functional Analysis, Differential Equations, Operator Theory, Banach Lattices, etc.

Note: This course is intended for senior undergraduate and graduate in Pure and Applied Mathematics.

Introduction to Hilbert Space

Going from finite to infinite dimension, Somebasic facts about vector spaces, Inner product, Normed spaces, Metric Spaces, Completeness, Complete spaces, Completion and closure, The Hilbert space L^2[a, b], The Banach space C[a,b], The Banach spaces L^p, Closed sets, Dense sets, Sets dense in the Hilbert space L^2, Polynomials are dense in the Banach space C[a,b], Hilbert Spaces, When does a norm come from an inner product?, The inner product is continuous, Orthonormal bases, Generalized Fourier series in L^2, Fourier series in L^2[a,b], Orthogonal polynomials, Orthogonal complements,The ProjectionTheorem,
Leasts quares approximation via subspaces, Linear operators in Hilbertspaces, Shift operators, Unitary operators, Isomorphic Hilbert spaces, Integral operators, Differential operators in L^2[a,b], A second order boundary value problem, General secondorder self-adjoint problems.

Note: This course is intended for Pure and Applied Mathematics students. Statistics and Enginnering students and the students in some other interdiciplinary programs are also welcome.

Banach Algebras

Banach algebras. Spectrum of a Banach algebra element. Gelfand theory of commutative Banach algebras. Analytic functional calculus. Hilbert space operators. C*-algebras of operators. Commutative C*-algebras. Spectral theorem for bounded self-adjoint and normal operators (both forms: the spectral integral and the "multiplication operator" formulation). Riesz theory of compact operators. Hilbert-Schmidt operators. Fredholm operators. The Fredholm index. Selected additional topics.

Note: This course is intended for Pure Mathematics Students.

Further Topics from Mathematics

An overview of the basic notions in multivariate calculus: vector functions and differentiation, curves and parametrization, functions of several variables, partial differentiation, differentiability, implicit functions, extreme values.

Mathematical Explorations

A mathematics appreciation course. Topics selected by the instructor to provide a contemporary mathematical perspective and experiences in mathematical thinking. May include historical material on the development of classical mathematical ideas as well as the evolution of recent mathematics.

Numbers and Proofs

A rigorous introduction to proof techniques and abstract mathematical reasoning with an emphasis on number systems: functions, sets and relations; the integers, prime numbers, divisibility and modular arithmetic; induction and recursion; real numbers; Cauchy sequences and completeness; complex numbers.

Linear Methods I

An introduction to systems of linear equations, vectors in Euclidean space and matrix algebra. Additional topics include linear transformations, determinants, complex numbers, eigenvalues, and applications.

Linear Methods II

An introductory course in the theory of abstract vector spaces: linear independence, spanning sets, basis and dimension; linear transformations and the rank-nullity theorem; the Gram-Schmidt algorithm and orthogonal diagonalization; singular value decomposition and other applications.

Linear Methods III

Canonical forms. Inner product spaces, invariant subspaces and spectral theory. Quadratic forms.

Curves and Surfaces

The fundamentals of the theory of curves and surfaces in three dimensional space. The theory of curves studies global properties of curves such as the four vertex theorem. The theory of surfaces introduces the fundamental quadratic forms of a surface, intrinsic and extrinsic geometry of surfaces, and the Gauss-Bonnet theorem.

Introduction to Mathematical Finance

An introduction to the fundamental concepts of mathematical finance in an elementary setting. Topics include: risk, return, no arbitrage principle; basic financial derivatives: options, forwards and future contracts; risk free assets, time value of money, zero coupon bonds; risky assets, binomial tree model, fundamental theorem of asset pricing; portfolio management and capital asset pricing model; no arbitrage pricing of financial derivatives; hedging.

Stochastic Calculus for Finance

Martingales in discrete and continuous time, risk-neutral valuations, discrete- and continuous-time (B,S)-security markets, Cox-Ross-Rubinstein formula, Wiener and Poisson processes, Ito formula, stochastic differential equations, Girsanov's theorem, Black-Scholes and Merton formulas, stopping times and American options, stochastic interest rates and their derivatives, energy and commodity models and derivatives, value-at-risk and risk management.

Advanced Mathematical Finance I

Lévy Processes (LP): fundamental concepts associated with LP such as infinite divisibility, the Lévy-Khintchine formula, the Lévy-Itô decomposition, subordinators, LP as time-changed Brownian motions, and also dealing with semi-groups and generators of LP, the Itô formula for LP, the Girsanov theorem, stochastic differential equations driven by LP, the Feynman-Kac formula, applications of LP and numerical simulation of LP.
Credit Risk: corporate bond markets, modelling the bankruptcy risk of a firm, and understanding how corporate bonds are priced. Stochastic Optimal Control and Applications in Finance: An introduction to the theory of stochastic optimal control and applications in finance and economics. Dynamic programming approach to optimal controls, solutions to several classes of typical optimal control problems, and application of the general theory to some classical models in finance and economics.

Advanced Mathematical Finance II

Monte Carlo Methods for Quantitative Finance: random number generation, simulation of stochastic differential equations, option valuation, variance reduction techniques, quasi-Monte Carlo methods, computing 'greeks', valuation of path-dependent and early-exercise options; applications to risk management; Markov Chain Monte Carlo methods. Energy, Commodity and Environmental Finance: energy and commodity markets; spot, futures, forwards and swap contracts; the theory of storage; stochastic models for energy prices; model calibration; emissions market modelling; weather derivatives; energy risk management; energy option valuation.
Advanced Topics in Mathematical Finance: An introduction to some of the main ideas in mathematical and

computational finance through a guided reading of some seminal papers from the last 100 years, starting with Bachelier's 1900 thesis, and including papers by Samuelson, Markowitz, Black & Scholes, Merton, Hull & White, Schwartz, Glasserman, and others.

Proofs and Modern Mathematics

How do mathematicians think? Why are the mathematical facts learned in school true? In this course students will explore higher-level mathematical thinking and will gain familiarity with a crucial aspect of mathematics: achieving certainty via mathematical proofs, a creative activity of figuring out what should be true and why. This course is ideal for students who would like to learn about the reasoning underlying mathematical results, but at a pace and level of abstraction not very intense, as a consequence benefiting from additional opportunity to explore the reasoning. Familiarity with one-variable calculus is strongly recommended at least at the AB level of AP Calculus since a significant part of the course develops some of the main results in that material systematically from a small list of axioms. We also address linear algebra from the viewpoint of a mathematician, illuminating notions such as fields and abstract vector spaces.

Modern Mathematics: Continuous Methods I

This is the first part of a theoretical (i.e., proof-based) sequence in multivariable calculus and linear algebra, providing a unified treatment of these topics. Covers general vector spaces, linear maps and duality, eigenvalues, inner product spaces, spectral theorem, metric spaces, differentiation in Euclidean space, submanifolds of Euclidean space as local graphs, integration on Euclidean space, and many examples. Necessary linear algebra contents are also covered. Students should know 1-variable calculus and have an interest in a theoretical approach to the subject. This series provides the necessary mathematical background for majors in all Computer Science, Economics, Mathematics, Mathematical and Computational Science, Natural Sciences, and Engineering.

Modern Mathematics: Discrete Methods I

This is the first part of a theoretical (i.e., proof-based) sequence in discrete mathematics and linear algebra. Covers general vector spaces, linear maps and duality, eigenvalues, inner product spaces, spectral theorem, counting techniques, and linear algebra methods in discrete mathematics including spectral graph theory and dimension arguments. The linear algebra content is covered jointly with Math 61CM. Students should have an interest in a theoretical approach to the subject. Prerequisite: score of 5 on the BC-level Advanced Placement calculus exam, or consent of the instructor.This series provides the necessary mathematical background for majors in Computer Science, Economics, Mathematics, Mathematical and Computational Science, and most Natural Sciences and some Engineering majors.

Modern Mathematics: Continuous Methods II

A proof-based introduction to manifolds and the general Stokes' theorem. This includes a treatment of multilinear algebra, further study of submanifolds of Euclidean space (with many examples), differential forms and their geometric interpretations, integration of differential forms, Stokes' theorem, and some applications to topology. Prerequisites: Modern Mathematics: Continuous Methods I.

Modern Mathematics: Discrete Methods II

This is the second part of a theoretical (proof-based) sequence with a focus on discrete mathematics. The central objects discussed in this course are finite fields. These are beautiful structures in themselves, and very useful in large areas of modern mathematics, and beyond. Our goal will be to construct these, understand their structure, and along the way discuss unexpected applications in combinatorics and number theory. Highlights of the course include a complete proof of a polynomial time algorithm for primality testing, Sidon sets and finite projective planes, and an understanding of a lovely magic trick due to Persi Diaconis. Prerequisite: Modern Mathematics: Continuous Methods I or Modern Mathematics: Discrete Methods I.

Modern Mathematics: Continuous Methods III

A proof-based course on ordinary differential equations. Topics include the inverse and implicit function theorems, implicitly-defined submanifolds of Euclidean space, linear systems of differential equations and necessary tools from linear algebra, stability and asymptotic properties of solutions to linear systems, existence and uniqueness theorems for nonlinear differential equations, behavior of solutions near an equilibrium point, and Sturm-Liouville theory. Prerequisite: Modern Mathematics: Continuous Methods I or Modern Mathematics: Discrete Methods I.

Modern Mathematics: Discrete Methods III

Third part of a proof-based sequence in discrete mathematics, though independent of the second part (62DM). The first half of the quarter gives a brisk-paced coverage of probability and random processes with an intensive use of generating functions and a rich variety of applications. The second half treats entropy, Bayesian inference, Markov chains, game theory, probabilistic methods in solving non-probabilistic problems. We use continuous calculus, e.g. in handling the Gaussian, but anything needed will be reviewed in a self-contained manner. Prerequisite: Modern Mathematics: Continuous Methods I or Modern Mathematics: Discrete Methods I.

Functions of a Real Variable

The development of 1-dimensional real analysis (the logical framework for why calculus works): sequences and series, limits, continuous functions, derivatives, integrals. Basic point set topology. Includes introduction to proof-writing.

Functions of a Complex Variable

Complex numbers, analytic functions, Cauchy-Riemann equations, complex integration, Cauchy integral formula, residues, elementary conformal mappings.

Stochastic Methods in Engineering

The basic limit theorems of probability theory and their application to maximum likelihood estimation. Basic Monte Carlo methods and importance sampling. Markov chains and processes, random walks, basic ergodic theory and its application to parameter estimation. Discrete time stochastic control and Bayesian filtering. Diffusion approximations, Brownian motion and an introduction to stochastic differential equations. Examples and problems from various applied areas. Prerequisites: exposure to probability and background in analysis.

Introduction to Stochastic Differential Equations

Brownian motion, stochastic integrals, and diffusions as solutions of stochastic differential equations. Functionals of diffusions and their connection with partial differential equations. Random walk approximation of diffusions. Introduction to stochastic control and Bayesian filtering.

Topics in Financial Math: Market microstructure and trading algorithms

Introduction to market microstructure theory, including optimal limit order and market trading models, Random matrix theory covariance models and their application to portfolio theory, Statistical arbitrage algorithms.

Mathematical Finance

Stochastic models of financial markets. Risk neutral pricing for derivatives, hedging strategies and management of risk, Multidimensional portfolio theory and introduction to statistical arbitrage.

Mathematical Modelling

This course is an introduction to mathematical modeling based on the use of elementary functions to describe and explore real-world phenomena and data. Linear, exponential, logarithmic, and polynomial function models are examined closely and are applied to real-world data in course assignments and projects. Other function models may also be considered. Throughout the course, computational tools (graphing calculators, spreadsheets, etc.) are used to implement, examine, and validate these models. Students are expected to actively engage in the modeling process by questioning phenomena, collecting or creating data, and using computational tools to develop their models and evaluate their efficacy.

Finite Mathematics I & II

Finite math is a branch of mathematics that deals with finite sets, or collections of things that have a fixed number of elements. One important concept is combinatorics, which is the study of the different ways that elements in a set can be combined or arranged. This area of math is often used in probability and statistics. Another key concept in finite math is graph theory. Graph theory looks at how different points (called "vertices") can be connected to one another with lines (called "edges"). Graph theory is used in many different fields, from computer science to engineering. Other topics that fall under the realm of finite math include matrix algebra, number theory, and logic. Matrix algebra looks at how numbers can be arranged in rows and columns to form a matrix, while number theory looks at the properties of numbers (like divisibility and prime factors). Logic, on the other hand, deals with the rules of reasoning and inference.

All of these concepts can be used in a variety of ways to solve real-world problems. For example, matrix algebra can be used to model traffic patterns or the behavior of electrical circuits. Graph theory can be used to model social networks or computer networks. Combinatorics can be used to optimize production processes or to predict the outcomes of sports games.

Special Functions

Covers in depth those functions which commonly occur in Physics and Engineering, namely, the Gamma, Beta, Bessel, Legendre, Hypergeometric, Hermite and Laguerre functions. Additional or alternative special functions may be included. Applications to Physics and Engineering will be discussed.

Integral Equations

In mathematics, integral equations are equations in which an unknown function appears under an integral sign. In this course the following topics will be included: Classification. Comparison of integral and differential equations. Green's functions. Analytical solutions of Fredholm equations with degenerate kernels. Approximate solutions of Frenhold equations with non-degenerate kernels. The Neumann series and resolvent kernels. Volterra equations of the second kind. Non-linear integral equations.

Calculus Revisited

A course for high school mathematics teachers. The course is built around a set of optimization problems, whose solution requires review of topics in first and second year calculus and linear algebra. Connections are made with topics in the Common Atlantic High School Mathematics Curriculum.

Mathematics of Financial Derivatives

Basics of options, futures, and other derivative securities. Introduction to Arbitrage. Brief introduction to partial differential equations. Stochastic calculus and Ito's Lemma. Option pricing using the Black-Scholes model. Put-call parity and Hedging. Pricing of European and American call and put options. Numerical methods for the Black-Scholes model: binary trees, moving boundary problems, and linear complementarity. The barrier, and other exotic options.

Real Analysis III

Review of measure theory; introduction to Hilbert and Banach spaces, Fourier transforms of L^P functions; Elementary Theory of Analytic Functions including Cauchy's Theorem and its consequences; Banach Algebras and applications.

Topcs in Complex Analysis

Complex Analytic Functions, Contour Integrals and Cauchy's Theorems; Taylor's, Laurent's, Liouville's and Rouche's Theorems, Residue Calculus, Riemann Mapping Theorem, Univalent Functions.

Introduction to Mathematical Modelling

Mathematical modelling is the process of describing a real world problem in mathematical terms, usually in the form of equations, and then using these equations both to help understand the original problem, and also to discover new features about the problem. Modelling both lies at the heart of much of our understanding of the world, and it allows engineers to design the technology of the future. With modelling we can travel to the edge of the universe, peer into the heart of the atom, and understand the future of our climate. This course includes the following topics:

- **Functions; Modeling with linear functions:** (Function definition; domain and range. Functions described by tables, graphs and formulas. Increasing and decreasing functions; local and absolute extrema. Concavity; inflection points. Average rate of change. Linear functions with applications. Slope-intercept and point-slope forms. Piecewise-linear functions with applications);
- **Linear regression; modeling with exponential functions: (**Fitting linear models to data. Evaluating model error; the sum of squared errors. Interpreting the correlation coefficient**.** Exponential growth functions with applications. Growth factors and rates. Doubling time. Compound interest. Exponential decay functions with applications. Decay factors and rates. Half-life);
- **Additional topics in exponential modeling, modeling with logarithmic functions; linear systems:** (Fitting exponential models to data. Continuous compounding. Continuous growth and decay. Newton's law of cooling and heating. Logarithmic functions with applications. Fitting logarithmic models to data. Matrices. Representing a system of linear equations with a matrix equation. Solving linear systems via matrix equations);
- **Modeling with polynomial functions:** (Quadratic functions with applications. Projectile motion. Maxima and minima applications. Fitting quadratic models to data. Interpreting the coefficient of determination. Polynomial functions of higher degree with applications. Polynomial interpolation. Fitting cubic and quartic models to data).

Advanced Mathematical Modelling

Countless phenomena in nature and society can be better understood by using mathematical tools and reasoning. Students in this course explore the myriad of ways in which mathematics can be applied. They analyze real-world situations, develop models which represent these situations mathematically, and check the models against reality. This course focuses on complex non-linear models. Students employ techniques from the theory of polynomial functions, systems of equations, geometry, trigonometry, and probability. They study linear programming, periodic phenomena such as biorhythms, problems faced by industry, and fractals and recursion theory, and complete projects such as designing and building a parabolic mirror. Students also delve into the history of applied mathematics. This course includes the following topics:

- **Partial differential equations (PDEs):** Revision of main solution techniques for elliptic and parabolic PDEs (separation of variables, Fourier transforms, similarity variables, Green's functions).

- **Introduction to applied stochastic methods:** Brownian motion and stochastic differential equations. Connection to PDEs: Fokker-Planck and backward Kolmogorov equation. Exit time problems.

- **Perturbation Methods:** Introduction to modelling concepts, dimensional analysis, perturbation techniques, matched asymptotics.

- **Application of Complex Variables.** Conformal mapping and applications. A selection from:(a) Hodograph and potential-plane techniques (b) Schwartz functions and vortex equilibria (c) Hele-Shaw free boundary problems (d) Two-dimensional freezing/melting problems.

- **Introduction to time series Analysis.** Forecasting—predicting future data based on historical data**.** Examples are: Weather prediction, Rainfall measurements, Temperature readings, Heart rate monitoring (EKG), Brain monitoring (EEG), Quarterly sales, Stock price prediction, Automated stock trading, Industry forecasts, Interest rate predictions