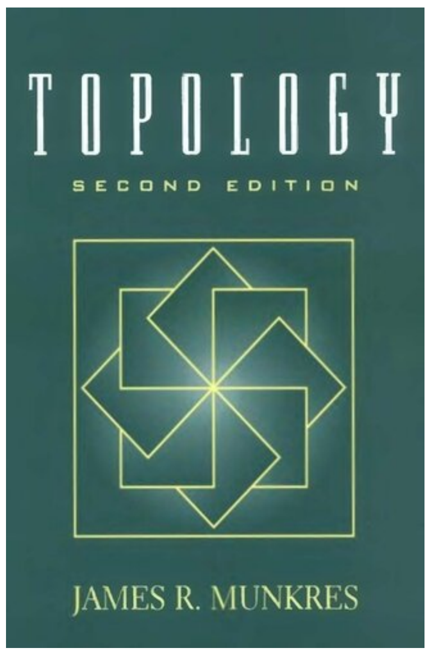
Course Description of General Topology

The subject of topoLogy is of interest in its own right, and it also serves to lay the foundations for future study in analysis, in geometry, and in algebraic topology. There is no universal agreement among mathematicians as to what a first course in topology should include; there are many topics that are appropriate to such a course, and not all are equally relevant to these differing purposes. In the choice of material to be treated, I have tried to strike a balance among the various points of view.

This course, consisting of the first eight chapters of the following book written by James R. Munkres, is devoted to the subject commonly called general topology. The first four chapters deal with the body of material that, in my opinion, should be included in any introductory topology course worthy of the name. This may be considered the "irreducible core" of the subject, treating as it does set theory, topological spaces, connectedness, compactness (through compactness of finite products), and the countability and separation axioms (through the Urysohn metrization theorem). The remaining four chapters of Part I explore additional topics; they are essentially independent of one another, depending on only the core material of Chapters 1-4. The instructor may take them up in any order he or she chooses.



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