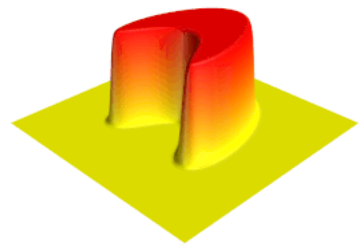
Partial differential equation (Course Description)

In mathematics, a **partial differential equation** (**PDE**) is an equation which computes a function between various partial derivatives of a multivariable function.



A visualisation of a solution to the two-dimensional heat equation with temperature represented by the vertical direction and color.

The function is often thought of as an "unknown" to be solved for, similar to how x is thought of as an unknown number to be solved for in an algebraic equation like $x^2 - 3x + 2 = 0$. However, it is usually impossible to write down explicit formulae for solutions of partial differential equations. There is correspondingly a vast amount of modern mathematical and scientific research on methods to numerically approximate solutions of certain partial differential equations using computers. Partial differential equations also occupy a large sector of pure mathematical research, in which the usual questions are, broadly speaking, on the identification of general qualitative features of solutions of various partial differential equations, such as existence, uniquity, regularity and stability. Among the many open questions are the existence and smoothness of solutions to the Navier–Stokes equations, named as one of the Millennium Prize Problems in 2000.

Partial differential equations are ubiquitous in mathematically oriented scientific fields, such as physics and engineering. For instance, they are foundational in the modern scientific understanding of sound, heat, diffusion, electrostatics, electrodynamics, thermodynamics, fluid dynamics, elasticity, general relativity, and quantum mechanics (Schrödinger equation, Pauli equation etc.). They also arise from many purely mathematical considerations, such as differential geometry and the calculus of variations; among other notable applications, they are the fundamental tool in the proof of the Poincaré conjecture from geometric topology.

Partly due to this variety of sources, there is a wide spectrum of different types of partial differential equations, and methods have been developed for dealing with many of the individual equations which arise. As such, it is usually acknowledged that there is no "general theory" of partial differential equations, with specialist knowledge being somewhat divided between several essentially distinct subfields.[1]

Ordinary differential equations form a subclass of partial differential equations, corresponding to functions of a single variable. Stochastic partial differential equations and nonlocal equations are, as of 2020, particularly widely studied extensions of the "PDE" notion. More classical topics, on which there is still much active research, include elliptic and parabolic partial differential equations, fluid mechanics, Boltzmann equations, and dispersive partial differential equations.

Table of Contents is in the next pages

Partial Differential Equations

Victor Ivrii

Department of Mathematics,

University of Toronto

Contents