Course Description of
Probability and its Applications

Probability means possibility. It is a branch of mathematics that deals with the occurrence of a random event. The value is expressed from zero to one. Probability has been introduced in Maths to predict how likely events are to happen. The meaning of probability is basically the extent to which something is likely to happen. This is the basic probability theory, which is also used in the probability distribution, where you will learn the possibility of outcomes for a random experiment.

In mathematics, ***Probability*** is a measure of the likelihood of an event to occur. Many events cannot be predicted with total certainty. We can predict only the chance of an event to occur i.e. how likely they are to happen, using it.

In this course will give you the tools needed to understand data, science, philosophy, engineering, economics, and finance. You will learn not only how to solve challenging technical problems, but also how you can apply those solutions in everyday life. With examples ranging from medical testing to sports prediction, you will gain a strong foundation for the study of statistical inference, stochastic processes, randomized algorithms, and other subjects where probability is needed.

How hard is probability for you?

Overall, probability tends to be a tricky class for most students when taking it for the first time. This is usually because it can be easy to think that you have calculated the probability of an event when it is not actually the true probability.

There are actually a number of factors that will determine how hard probability is for you and there are a number of things that you can do to make it easier.

- In probability, you will be learning about things such as basic combinatorics which involve sets and counting problems, less complex probability problems, expect value problems and problems that use probability distributions. There are a lot of things to learn in probability and this can cause difficulty for some students.
- In addition, the problems can also be confusing. Often it will seem like you have calculated the correct probability for a problem when, in fact, you have not. The best way to prevent this is to work through lots of problems until you are able to see the correct method of solving common problems.
- Also, probability does involve a lot of math. Depending on the prerequisites for the class, it will also make use of a lot of calculus and multi-variate calculus. The calculus problems tend to be of a similar difficulty to what you would find in a typical calculus class. If your class does not involve calculus, it will still likely involve a fair amount of mathematics so you will still need to study more than most other classes.
- How difficult probability and most other college classes will be for you will depend largely on the professor that you take the class with. The professor will usually dictate the pace of the class, the scope of the exams, some will tell you to know everything from the text and others will give you a study guide.
- It will also depend a lot on your own background, if you have taken lots of math classes before and have done well in them, you will probably do well in probability as well.

Even though there are a number of reasons why probability can be a difficult course, there are a number of things that can make it less difficult.

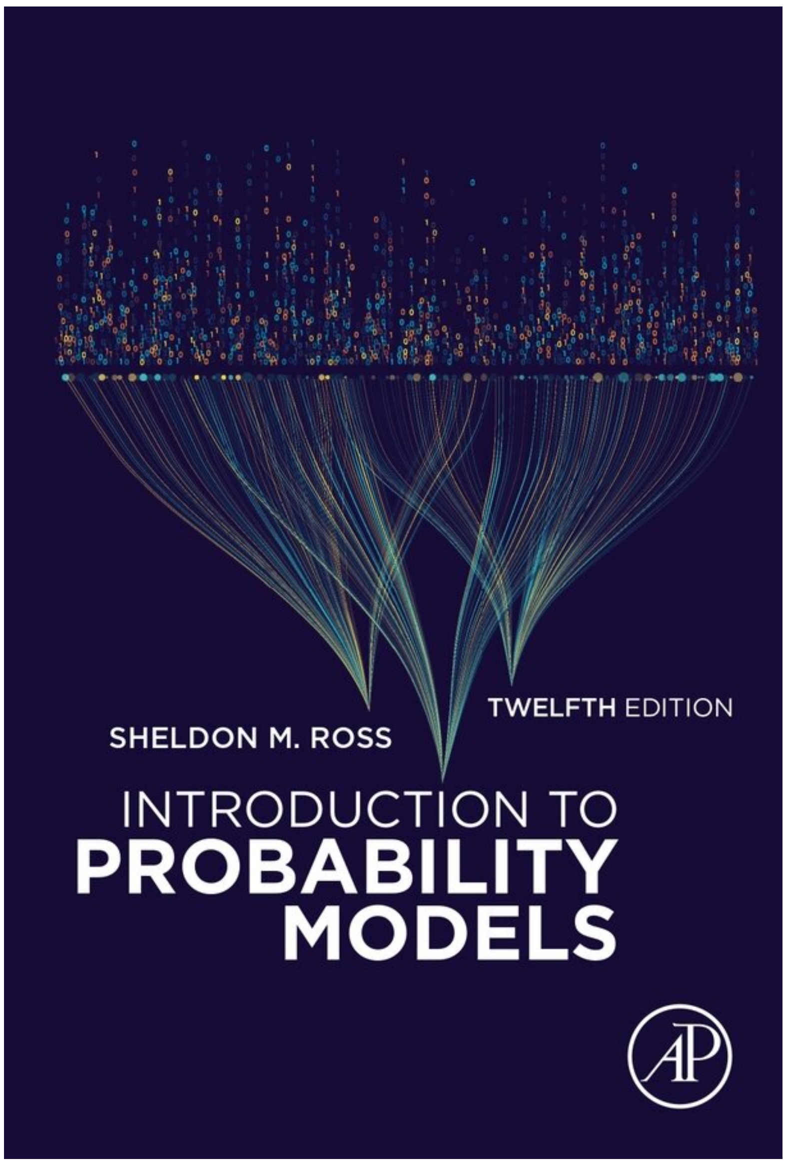
- First, probability is a topic that has clear real-world use cases. Being able to see the usefulness of the course usually helps to motivate students to study for the course and to do well in it.
- Also, unlike other math-based topics, introductory probability is not usually proof heavy. This is good for most undergraduate and high school students since proof heavy contents tend to be the harder contents.
- The most important thing that will greatly improve your probability learning much easier, would be to prepare for it ahead of time. You will personally make probability less difficult, if you put most of it down to preparing for the course before actually taking it.

Recommended Text Books

According to a survey report, several students voted that mathematics is one of the toughest subjects, and probability is considered to be a complicated topic in which most of the students get puzzled. Therefore, we have analyzed that students need some kind of suggestions that can help the students to deal with probability problems. Here, we have listed some of the probability books that can help out the students.

- *An Introduction to Probability Theory and Its Applications: By William Feller* (1991)
- *The Probability Tutoring Book: By Carole Ash, Published By IEEE Press (1993)*
- *Introduction to Probability Models, Twelfth Edition, Sheldon M. Ross, Published By AP. (2019)*
- *A First Course in Probability, By Sheldon Ross, Global Edition, (2020)*

NOTE: In this website our tutors teach the contents of the text book *"Introduction to Probability Models, Twelfth Edition, Sheldon M. Ross, Published By AP. (2019)"* to their students. The Preface and Table of contents of this book Are given in the next pages.



Preface

This text is intended as an introduction to elementary probability theory and stochastic processes. It is particularly well suited for those wanting to see how probability theory can be applied to the study of phenomena in fields such as engineering, computer science, management science, the physical and social sciences, and operations research.

It is generally felt that there are two approaches to the study of probability theory. One approach is heuristic and nonrigorous and attempts to develop in the student an intuitive feel for the subject that enables him or her to "think probabilistically." The other approach attempts a rigorous development of probability by using the tools of measure theory. It is the first approach that is employed in this text. However, because it is extremely important in both understanding and applying probability theory to be able to "think probabilistically," this text should also be useful to students interested primarily in the second approach.

New to This Edition

The twelfth edition includes new text material, examples, and exercises in almost every chapter. Newly added Sections begin in Chapter 1 with Section 1.7, where it is shown that probability is a continuous function of events. The new Section 2.8 proves the Borel–Cantelli lemma and uses it as the basis of a proof of the strong law of large numbers. Subsection 5.2.5 introduces the Dirichlet distribution and details its relationship to exponential random variables. Notable also in Chapter 5 is a new approach for obtaining results for both stationary and non-stationary Poisson processes. The biggest change in the current edition, though, is the addition of Chapter 12 on coupling methods. Its usefulness in analyzing stochastic systems is indicated throughout this chapter.

Course

Ideally, this text would be used in a one-year course in probability models. Other possible courses would be a one-semester course in introductory probability theory (involving Chapters 1–3 and parts of others) or a course in elementary stochastic processes. The textbook is designed to be flexible enough to be used in a variety of possible courses. For example, I have used Chapters 5 and 8, with smatterings from Chapters 4 and 6, as the basis of an introductory course in queueing theory.

Examples and Exercises

Many examples are worked out throughout the text, and there are also a large number of exercises to be solved by students. More than 100 of these exercises have been starred and their solutions provided at the end of the text. These starred problems can be used for independent study and test preparation. An Instructor's Manual, containing solutions to all exercises, is available free to instructors who adopt the book for class.

Organization

Chapters 1 and 2 deal with basic ideas of probability theory. In Chapter 1 an axiomatic framework is presented, while in Chapter 2 the important concept of a random variable is introduced. Section 2.6.1 gives a simple derivation of the joint distribution of the sample mean and sample variance of a normal data sample. Section 2.8 gives a proof of the strong law of large numbers, with the proof assuming that both the expected value and variance of the random variables under consideration are finite.

Chapter 3 is concerned with the subject matter of conditional probability and conditional expectation. "Conditioning" is one of the key tools of probability theory, and it is stressed throughout the book. When properly used, conditioning often enables us to easily solve problems that at first glance seem quite difficult. The final section of this chapter presents applications to (1) a computer list problem, (2) a random graph, and (3) the Polya urn model and its relation to the Bose–Einstein distribution. Section 3.6.5 presents k-record values and the surprising Ignatov's theorem.

In Chapter 4 we come into contact with our first random, or stochastic, process, known as a Markov chain, which is widely applicable to the study of many real-world phenomena. Applications to genetics and production processes are presented. The concept of time reversibility is introduced and its usefulness illustrated. Section 4.5.3 presents an analysis, based on random walk theory, of a probabilistic algorithm for the satisfiability problem. Section 4.6 deals with the mean times spent in transient states by a Markov chain. Section 4.9 introduces Markov chain Monte Carlo methods. In the final section we consider a model for optimally making decisions known as a Markovian decision process.

In Chapter 5 we are concerned with a type of stochastic process known as a counting process. In particular, we study a kind of counting process known as a Poisson process. The intimate relationship between this process and the exponential distribution is discussed. New derivations for the Poisson and nonhomogeneous Poisson processes are discussed. Examples relating to analyzing greedy algorithms, minimizing highway encounters, collecting coupons, and tracking the AIDS virus, as well as material on compound Poisson processes, are included in this chapter. Section 5.2.4 gives a simple derivation of the convolution of exponential random variables.

Chapter 6 considers Markov chains in continuous time with an emphasis on birth and death models. Time reversibility is shown to be a useful concept, as it is in the study of discrete-time Markov chains. Section 6.8 presents the computationally important technique of uniformization.

Chapter 7, the renewal theory chapter, is concerned with a type of counting process more general than the Poisson. By making use of renewal reward processes, limiting results are obtained and applied to various fields. Section 7.9 presents new results concerning the distribution of time until a certain pattern occurs when a sequence of independent and identically distributed random variables is observed. In Section 7.9.1, we show how renewal theory can be used to derive both the mean and the variance of the length of time until a specified pattern appears, as well as the mean time until one of a finite number of specified patterns appears. In Section 7.9.2, we suppose that the random variables are equally likely to take on any of m possible values, and compute an expression for the mean time until a run of m distinct values occurs. In Section 7.9.3, we suppose the random variables are continuous and derive an expression for the mean time until a run of m consecutive increasing values occurs.

Chapter 8 deals with queueing, or waiting line, theory. After some preliminaries dealing with basic cost identities and types of limiting probabilities, we consider exponential queueing models and show how such models can be analyzed. Included in the models we study is the important class known as a network of queues. We then study models in which some of the distributions are allowed to be arbitrary. Included are Section 8.6.3 dealing with an optimization problem concerning a single server, general service time queue, and Section 8.8, concerned with a single server, general service time queue in which the arrival source is a finite number of potential users.

Chapter 9 is concerned with reliability theory. This chapter will probably be of greatest interest to the engineer and operations researcher. Section 9.6.1 illustrates a method for determining an upper bound for the expected life of a parallel system of not necessarily independent components and Section 9.7.1 analyzes a series structure reliability model in which components enter a state of suspended animation when one of their cohorts fails.

Chapter 10 is concerned with Brownian motion and its applications. The theory of options pricing is discussed. Also, the arbitrage theorem is presented and its relationship to the duality theorem of linear programming is indicated. We show how the arbitrage theorem leads to the Black–Scholes option pricing formula.

Chapter 11 deals with simulation, a powerful tool for analyzing stochastic models that are analytically intractable. Methods for generating the values of arbitrarily distributed random variables are discussed, as are variance reduction methods for increasing the efficiency of the simulation. Section 11.6.4 introduces the valuable simulation technique of importance sampling, and indicates the usefulness of tilted distributions when applying this method.

Chapter 12 introduces the concept of coupling and shows how it can be effectively employed in analyzing stochastic systems. Its use in showing stochastic order relations between random variables and processes—such as showing that a birth and death process is stochastically increasing in its initial state—is illustrated. It is also shown how coupling can be of use in bounding the distance between distributions, in obtaining stochastic optimization results, in bounding the error of Poisson approximations, and in other areas of applied probability.

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