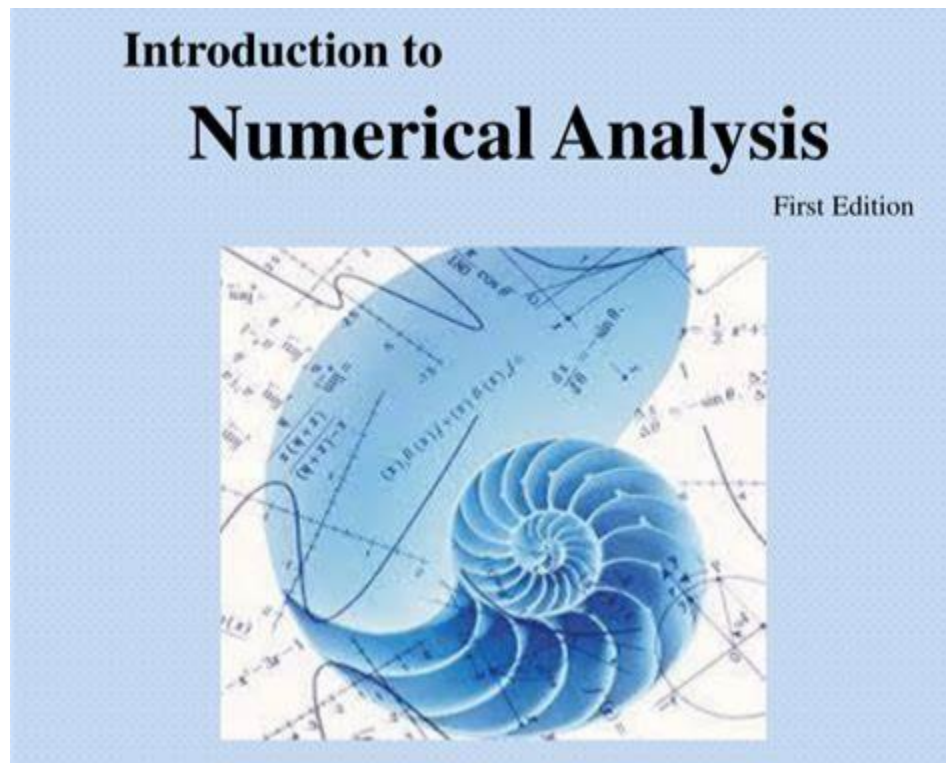


# Elementary Numerical Analysis

The field of numerical analysis is, broadly speaking, is concerned with obtaining approximate solutions to mathematical problems that can be implemented on a computer. The theory of approximation can be surprisingly deep and elegant, given the messiness of the problems it seeks to solve. Under the wide umbrella of the subject is both pure analysis and more practical computational work. Some examples include:

- Theory (the analysis)
  - Convergence (limits of sequences that approach the true solution)
  - Finite-dimensional spaces for approximation
  - Discrete analogues of continuous processes
- Applied (some where in between)
  - Derivation of (practical) numerical methods
  - Intuition for interpreting results, measuring error
  - Adapting / generalizing methods to get desired properties
- Implementation (the numerical)
  - Translating methods to actual code
  - Efficient implementation
  - Developing packages for computing (COMSOL, Matlab, Rtc.)

In this course, we focus more on the first two aspects and address the last one in less depth. Hopefully, you will be convinced by the end that an understanding of the underlying mathematics is invaluable, even when one is concerned with practical results.



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