Courses in Application of Mathematics

The Following is the list of all Courses in Application of Mathematics that TMS can offer either online or home tutoring

Applied Calculus: for the Managerial, Life, and Social Sciences

Applied Calculus is exciting and informative. The course focuses on making calculus easy to learn with clear writing, effective integration of technology tools, and relevant applications. The contents make a wonderful introductions to applied calculus, and the extra sessions for solving exercises help students monitor their comprehension and progress. In this course, the sessions will be organized in such a way to better suit motivated students. In the market-leading Calculus for the Managerial, Life and Social Sciences, TMS provides an accurate, accessible presentation of calculus combined with just the right balance of applications, pedagogy, and technology to help students succeed in the course. The contents of this course are exactly those contents that are listed in the book "Applied Calculus for the Managerial, Life, and Social Sciences_ A Brief Approach-Cengage Learning (2014)", written by Soo T. Tan.

Applied Calculus for Engineers I, II, III, & IV

Engineering is defined as "the profession in which a knowledge of the mathematical and natural sciences gained by study, experience, and practice is applied with judgment to develop ways to utilize, economically, the materials and forces of nature for the benefit of mankind." Some engineers directly use calculus in their daily practice and some use computer programs based on calculus that simplify engineering design. Two methods of calculus, differentiation and integration, are particularly useful in the practice of engineering, and are generally used for optimization and summation, respectively.

- In Civil Engineering many aspects require calculus. Firstly, derivation of the basic fluid mechanics equations requires calculus. For example, all hydraulic analysis programs, which aid in the design of storm drain and open channel systems, use calculus numerical methods to obtain the results. In hydrology, volume is calculated as the area under the curve of a plot of flow versus time and is accomplished using calculus.
- In structural engineering, calculus is used to determine the forces in complex configurations of structural elements. Structural analysis relating to seismic design requires calculus. In a soil structure context, calculations of bearing capacity and shear strength of soil are done using calculus, as is the determination of lateral earth pressure and slope stability in complex situations.
- Many examples of the use of calculus are found in mechanical engineering, such as computing the surface area of complex objects to determine frictional forces, designing a pump according to flow rate and head, and calculating the power provided by a battery system. Newton's law of cooling is a governing differential equation in HVAC design that requires integration to solve.
- Numerous examples of the use of calculus can be found in aerospace engineering. Thrust over time calculated using the ideal rocket equation is an application of calculus. Analysis of rockets that function in stages also requires calculus, as does gravitational modeling over time and space. Almost all physics models, especially those of astronomy and complex systems, use some form of calculus.
- Numerous examples of the use of calculus can also be found in other branches of Engineering which are too vast and we are not going to explain them here.

The following describes the contents that will be included in these four courses:

- Applied Calculus for Engineers I treats basic calculus with differential and integral calculus of single valued functions. We use a systematic approach following a bottom-up strategy to introduce the different terms needed.

- Applied Calculus for Engineers II covers series and sequences and first order differential equations as a calculus part. The second part of the this course is related to linear algebra.
- Applied Calculus for Engineers III treats vector calculus and differential equations of higher order.
- Applied Calculus for Engineers IV uses the material of the previous courses in numerical applications; it is related to numerical methods and practical calculations.

As prerequisites It is assumed that students had the basic high school education in algebra and geometry. However, the presentation of the material starts with the very elementary subjects like numbers and introduces in a systematic way step by step the concepts for functions. This allows us to repeat most of the material known from high school in a systematic way, and in a broader frame. This way the students will be able to use and categorize their knowledge and extend their old frame work to a new one The numerous examples from engineering and science stress on the applications in engineering. The idea behind these is summarized in a three step process: Theory ====> Examples ====> Applications.

When examples are discussed in connection with the theory, then it turns out that the theory is not only valid for this specific example but also useful for a broader application. In fact usually theorems or a collection of theorems can even handle whole classes of problems. These classes are sometimes completely separated from this introductory example; e.g. the calculation of areas to motivate integration or the calculation of the power of an engine, the maximal height of a satellite in space, the moment of inertia of a wheel, or the probability of failure of an electronic component. All these problems are solvable by one and the same method, integration.

However, the three step process is not a feature which is always used. Some times we have to introduce mathematical terms which are used later on to extend our mathematical frame. This means that the courses are not organized in a historic sequence of facts as traditional mathematics texts. We introduce definitions, theorems, and corollaries in a way which is useful to create progress in the understanding of relations.

By offering these four courses, We hope that we have created a coherent way of a first approach to Calculus for engineers.

Linear Algebra and its Applications

This course presents linear algebra as the theory and practice of linear spaces and linear maps with a unique focus on the analytical aspects as well as the numerous applications of the subject. In addition to thorough coverage of linear equations, matrices, vector spaces, game theory, and numerical analysis, this course offers extra sessions that enhance the course's accessibility, including expanded topical coverage in the early sessions, additional exercises, and solutions to selected problems. Beginning sessions are devoted to the abstract structure of finite dimensional vector spaces, and subsequent chapters address convexity and the duality theorem as well as describe the basics of normed linear spaces and linear maps between normed spaces. Further updates and revisions have been included in these extra sessions to reflect the most up-to-date coverage of the topic, including:The QR algorithm for finding the eigenvalues of a self-adjoint matrix, The Householder algorithm for turning self-adjoint matrices into tridiagonal form, and The compactness of the unit ball as a criterion of finite dimensionality of a normed linear space. Additionally, eight new topics such as: the Fast Fourier Transform; the Spectral Radius Theorem; the Lorentz Group; the Compactness Criterion for Finite Dimensionality; the Characterization of Commentators; Proof of Liapunov's Stability Criterion; the Construction of the Jordan Canonical Form of Matrices; and Carl Pearcy's elegant proof of Halmos' Conjecture about the Numerical Range of matrices have been added. Clear, concise, and superbly organized, this course serves as an excellent choice for advanced undergraduate and graduate level students. Its comprehensive treatment of the subject also makes it an ideal course for industry professionals.

Complex Analysis and its Applications

Complex Analysis with Applications in Science and Engineering weaves together theory and extensive applications in mathematics, physics and engineering. This course will be suitable for undergraduate and graduate students in the areas noted above. Key Features of this course: Excellent coverage of topics such as series, residues and the evaluation of integrals, multivalued functions, conformal mapping, dispersion relations and analytic continuation Systematic and clear presentation with many diagrams to clarify discussion of the material. Numerous worked examples and a large number of assigned problems. The course includes several excursions into applications of interest to physicists and electrical engineers, as circuit analysis and an extensive session on dispersion relations. The course would be of particular interest to physics and electrical engineering students. Mathematicians might find it a good hunting ground for offbeat approaches to familiar themes and for various other serendipities. This course might be useful for those who are still familiar with complex analysis and who are searching for several examples, exercises or applications in physics and engineering. Contents include:

- A Brief History of Real numbers, Complex numbers, and Argand diagram;
- Complex Numbers: (Algebra of complex numbers, Cartesian, Trigonometric and Polar Forms of complex numbers, nth root of a complex number (specially roots of unity), Complex Numbers and AC or DC circuits with resistors;
- Complex Variables: (Derivatives and the Cauchy-Riemann conditions, Analyticity, Laplace's equation for an analytic function, Integrals of Analytic Functions, Singularities, Removable and nonremovable singularities, Cauchy's Residue Theorems);
- Series, Limits and Residues: (Taylor Series for Analytic Functions, Laurent Series for a Singular function or an analytic function or a function with a pole of order M, Convergence and The Cauchy Ratio, Limits and Series, Arithmetic Combinations of Power Series, Three methods for determining the residue);
- Evaluation of Integrals: (Integrals Along The Entire Real Axis, Integrals of Functions of $\sin\theta$ and $\cos\theta$, Cauchy's Principal Value Integral and The Dirac δ Symbol, Miscellaneous Integrals);
- Multivalued Functions, Branch Points and Cuts: (Non-Integer Power, Logarithm Functions, Riemann Sheets, Branch Points and Cuts, Branch Structure, Multiple Branch Points, Evaluation of Integrals, Specific examples);
- Singularities of Functions Defined by Integrals: (The Integrand is Analytic, The Integrand is Singular, Limits of The Integral are Variable);
- Conformal Mappings: (Properties of a Mapping, Linear and Bilinear Transformations, Schwarz-Christoffel Transformation, Laplace's equation, Boundary conditions, Applications of conformal mapping to problems in electrostatics);
- Dispersion Relations: (Kramers-Kronig Dispersion Relations Over, Kramers-Kronig Dispersion Relations Over Half, Dispersion Relations for a Function, Subtracted Dispersion Relations, Dispersion Relations and a Representation of The Dirac δ-Symbol);
- Analytic Continuation: (Analytic Continuation by Series, Analytic Continuation of The Factorial, The gamma function, Half integers factorial, Approximation of $z!$ for $Re(z)$ positive and large, Approximation of $z!$ for $|z|$ small.

Fourier Analysis and its Applications

This course presents the theory and applications of Fourier series and integrals, eigenfunction expansions, and related topics, on a level suitable for advanced undergraduates. It includes material on Bessel functions, orthogonal polynomials, and Laplace transforms, and it concludes with chapters on generalized functions and Green's functions for ordinary and partial differential equations. The course deals almost exclusively with aspects of these subjects that are useful in physics and engineering, and includes a wide variety of applications. On the theoretical side, it uses ideas from modern analysis to develop the concepts and reasoning behind the techniques without getting bogged down in the technicalities of rigorous proofs.
In this course the fo;;owing topics will be included:

- Basic convergence theorem of Fourier series and its applications
- Mean convergence
- Cesaro summability
- Fejer's theorem and its applications

- The applications of Cesaro summability and Fejer's theorem;
- The space S of rapidly decreasing functions;
- Fourier transform as an operator from S to S and the Plancherel's theorem;
- Sturm-Liouville problems and its applications;
- The diverse applications of Fourier Analysis, specially Fourier Transform, in a descriptive way.

Applied Functional Analysis

There are two different ways of teaching mathematics, namely, (i) the systematic way, and (ii) the application-oriented way. More precisely, by (i), we mean a systematic presentation of the material governed by the desire for mathematical perfection and completeness of the results. In contrast to (i), approach (ii) starts out from the question "What are the most important applications?" and then tries to answer this question as quickly as possible. Here, one walks directly on the main road and does not wander into all the nice and interesting side roads. The present course is based on the second approach. It is addressed to undergraduate and beginning graduate students of mathematics, physics, and engineering who want to learn how functional analysis elegantly solves mathematical problems that are related to our real world and that have played an important role in the history of mathematics. The students should sense that the theory is being developed, not simply for its own sake, but for the effective solution of concrete problems. In this course the following topics will be included:

- Introduction: (Sets, Sequences, Real Numbers, Complex Numbers, Linear Spaces);
- Metric Spaces: (Definitions, Completeness, Compact Sets, Continuous Functions on Compact Sets, The Banach Contraction Principle);
- The Lebesgue Integral and Lp Spaces: (Measurable Sets in R^k, Measurable Functions, The Lebesgue Integral Lp Spaces);
- Continuous Linear Operators and Functionals: (Definitions, Examples, Operator Norm, Main Principles of Functional Analysis, Compact Linear Operators, Linear Functionals, Dual Spaces, Weak Topologies);
- Distributions, Sobolev Spaces: (Test Functions, Friedrichs' Mollification, Scalar Distributions, Some Operations with Distributions, Convergence in Distributions, Differentiation of Distributions, Differential Equations for Distributions, Sobolev Spaces, Bochner's Integral, Vector Distributions,W_(m ,p) (a ,b ;X) Spaces);
- Hilbert Spaces: (Examples, Jordan–von Neumann Characterization Theorem, Projections in Hilbert Spaces The Riesz Representation Theorem, Lax–Milgram Theorem, Fourier Series Expansions);
- Adjoint, Symmetric, and Self-adjoint LinearOperators: (The Adjoint of a Linear Operator, Adjoints of Operators on Hilbert Spaces, The Case of Compact Operators, Symmetric Operators and Self-adjoint Operators);
- Eigenvalues and Eigenvectors: (Definition and Examples, Main Results, Eigenvalues of $-\Delta$ Under the Dirichlet Boundary Condition, Eigenvalues of $-\Delta$ Under the Robin BoundaryCondition, Eigenvalues of $-\Delta$ Under the Neumann BoundaryCondition, Some Comments);
- Semigroups of Linear Operators: (Definitions, Some Properties of C_0 -Semigroups, Uniformly Continuous Semigroups, Groups of Linear Operators, Definitions and Link to Operator Semigroups, Translation Semigroups,The Hille–Yosida Generation Theorem,The Lumer–Phillips Theorem, The Feller–Miyadera–Phillips Theorem, A Perturbation Result, Approximation of Semigroups, The Inhomogeneous Cauchy Problem, Applications, The Heat, The Wave, and The Transport Equation, The Telegraph System);
- Solving Linear Evolution Equationsby the Fourier Method: (First Order Linear Evolution Equations, Second Order Linear Evolution Equations, Examples);
- Integral Equations: (Volterra Equations, Fredholm Equations).

Application of Fast Fourier Transforms

This course presents an introduction to the principles of the Fast Fourier Transform. This course covers FFTs, frequency domain filtering, and applications to video and audio signal processing. As fields like communications, speech and image processing, and related areas are rapidly developing, the FFT as one of essential parts in digital signal processing has been widely used. This course provides thorough and detailed explanation of important or up-to-date FFTs. It also has adopted modern approaches like MATLAB examples and projects for better understanding of diverse FFTs. The following topics will be included in this course:

- Various applications of the Discrete Fourier Transform (DFT);
- Introductory material on the properties of the DFT for the equally spaced samples;
- Fast algorithms to be mainly categorized as Decimation-In-Time (DIT) or Decimation-In-Frequency (DIF) approaches;
- Fast algorithms like Split-Radix, Winograd Algorithm and others;
- Integer Fast Fourier Transform (FFT) which approximates the DFT.
- Extension of One-dimensional DFT to the two-dimensional and multi-dimensional signals.
- Applications to filtering;
- Variance Distribution in the DFT domain;
- Introducing the diagonalization of a circulant matrix using the DFT matrix;
- Fast algorithms for the two dimensional DFT;
- Introductory material on the properties of Nonuniform DFT (NDFT) for the none-qually spaced samples.
- Numerous applications of the FFT;
- Performance comparison of discrete transforms;
- Spectral distance measures of image quality;
- Integer Descrete Cosine Transforms;
- Descrete Cosine and Sine Transforms;
- A briefly covering of Kronecker products and separability;
- Numerous problems and projects will be listed during the course.

Variational Analysis and its Applications

Variational analysis, as now understood, is a relatively young area of mathematics. From one side, it can be viewed as an outgrowth of the calculus of variations, constrained optimization, and optimal control, and also of variational principles in mathematical physics and mechanics that go back to the 18th century. On the other hand, modern variational principles and techniques are largely based on perturbations, approximations, and the (unavoidable) usage of generalized differentiation. All of this requires developing new forms of analysis and thus manifests the creation of a new discipline in mathematics that strongly combines and unifies analytic and geometric ideas.

This course presented to the students' attention pursues two important goals. The first goal is to give a systematic and easily understandable exposition of the key concepts and facts of variational analysis with selected applications in finite-dimensional spaces. Another goal of this course is encouraging the interested students to learn more on variational analysis and to develop their research skills in this field by performing (at least partly) the exercises presented after the basic material of each chapter.

Topics included in this course are the following:

- Constructions of Generalized Differentiation;
- Fundamental Principles of VariationalAnalysis;
- Well-Posedness and Coderivative Calculus;
- First-Order Subdifferential Calculus;
- Coderivatives of Maximal Monotone Operators;
- Nondifferentiable and Bilevel Optimization;
- Semi-infinite Programs with Some Convexity;
- Nonconvex Semi-infinite Optimization;
- Variational Analysis in Set Optimization;
- Set-Valued Optimization and Economics;

Measure Theory and its Applications

Measure theory has numerous applications in real-world fields such as physics, engineering, economics, and finance, amongst others. It also has essential applications in a wide variety of fields, including Functional Analysis, Harmonic Analysis, and Probability Theory.Techniques from the field of measure theory are hence essential for the work of any mathematician. In this course we will cover the following topics along with their applications in real world life and in theoretical context:

- Introduction and Motivation of Measure theory, Jordan measurability and Jordan content:
- Basic properties of Jordan content and connection with Riemann integrals, Motivation and definition of Lebesgue outer measure on R^n;
- Properties of Lebesgue outer measure on R^n, Caratheodory extension theorem;
- Lebesgue measurability, Vitali and Cantor sets, Boolean and sigma algebras;
- Abstract measure spaces with examples: Borel and Radon measures, Metric outer measures, Lebesgue-Stieljes measures, Hausdorff measures and dimension (extra content);
- Measurable functions and abstract Lebesgue integration, Monotone convergence theorem, Fatou's lemma, Tonnelli's theorem;
- Borel-Cantelli Lemma, Dominated convergence theorem, the space L^1;
- Various modes of convergence and their inter-dependence;
- Riesz Representation Theorem (RRT), examples and application of measures constructed via RRT;
- Product measures and Fubini-Tonnelli theorem;
- Hardy-Littlewood Maximal inequality and Lebesgue's differentiation theorem;
- Lebesgue's differentiation theorem (continued);
- Examples of Various application of measure theory in Real-World Life and in Theoretical Contexts.

Operator Theory and its Applications

In mathematics, operator theory is the study of linear operators on function spaces, beginning with differential operators and integral operators. The operators may be presented abstractly by their characteristics, such as bounded linear operators or closed operators, and consideration may be given to nonlinear operators. The study, which depends heavily on the topology of function spaces, is a branch of functional analysis.
Topics in this course will be covered in two parts:

Part 1 is about the study of operators and includes the following contents:

- Review of Hilbert space Theory , Bounded operators on Hilbert spaces, examples;
- Adjoint an operator, examples, Self-adjoint, normal, positive, unitary, isometries, partial isometries:
- Orthogonal projections with examples, invariant subspaces, numerical range and characterization of operators;
- Banach Algebras, inertibility, spectrum;
- Gelfand-Mazur theorem, spectral radius formula, spectral mapping theorem;
- Subdivion of the spectrum of an operator, properties of the various spectra;
- Computing spectrum with examples;
- Existence of square root, polar-decomposition;
- Compact operators, properties;
- Spectral theorem for compact self-adjoint operators, spectral theorem for compact normal operators, Schmidt-decomposition, Monotone convergence theorem for self-adjoint operators;
- Spectral theorem for self-adjoint operators, continuous functional calculus, spectral theorem for self-adjoint operators(multiplication form);
- Spectral theorem for normal operators (both integral and multiplication form), continuous functional calculus for normal operators.

Part 2 is about the application of Operator Theory and it includes both expository essays and original research papers illustrating the diversity and beauty of insights gained by applying operator theory to concrete problems. The topics range from control theory, frame theory, Toeplitz and singular integral operators, Schrödinger, Dirac, and Kortweg-de Vries operators, Fourier integral operator zeta-functions, C*-algebras and Hilbert C*-modules to questions from harmonic analysis, Monte Carlo integration, Fibonacci Hamiltonians, and many more. The course also offers students in operator theory open problems from applications that might stimulate their work and shows those from various applied fields, such as physics, engineering, or numerical mathematics how to use the potential of operator theory to tackle interesting practical problems.