Course Description of Real and Complex Analysis

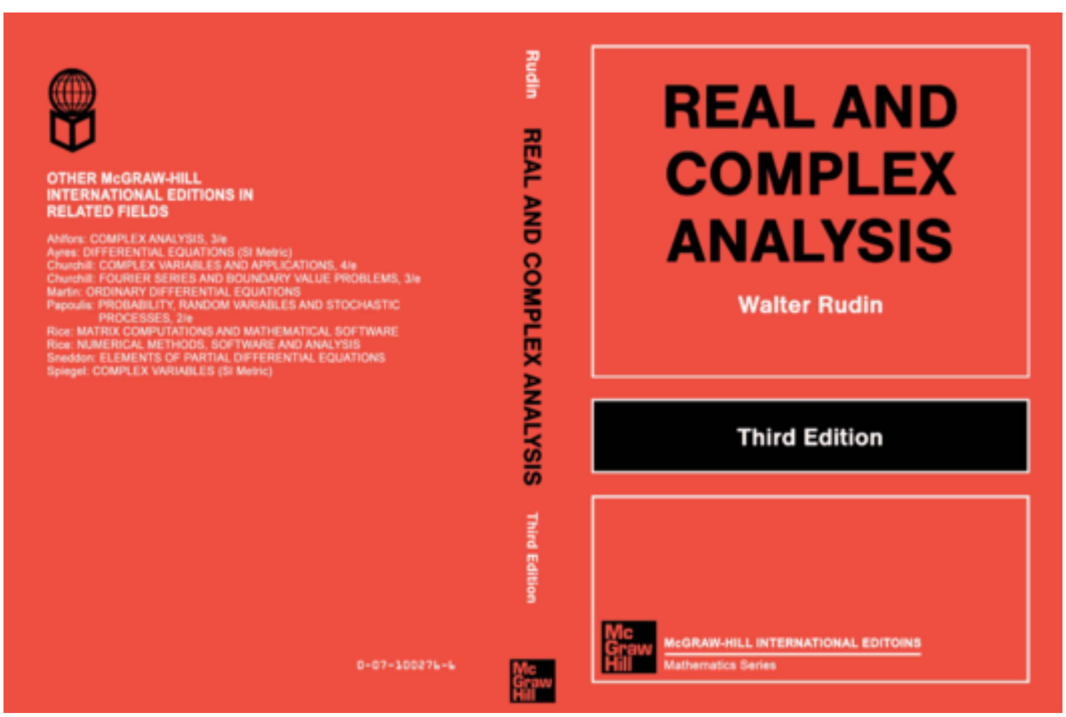
This course covers many interesting things, and proofs will be given in almost all sessions. The sessions on Banach algebras is a gem; this subject combines algebra, analysis, and topology, and the exposition shows clearly how the three areas work together. This course show that real and complex analysis should be studied together rather than as two subjects, and to give a a modern treatment.

The first third of the course is devoted to measure and integration. The presentation is based on measures on abstract spaces with σ-algebras. It includes brief introductions to Hilbert space and Banach spaces, with material that will be used in the complex-variables proofs later. This is the only part of the course that deals with spaces more general than the real line and the complex plane, however it's not any harder than it would be if we stuck to the real line. This part also includes differentiation (of measures) and product spaces (i.e., the Fubini theorem). The rest of the course is about analysis on the complex plane. It starts with a short discussion on Fourier transforms, then presents some sessions in complex variables that is traditional in terms of the theorems proved, but has very slick proofs using what has gone before. The traditional part ends with the little Picard theorem. The last quarter of the course consists of several short chapters on advanced topics in complex analysis; these include Hp spaces, Banach algebras, holomorphic Fourier transforms, and a characterization of functions that are the uniform limit of polynomials (Mergelyan's theorem).

The approach is not very concrete; there are very few worked examples (many of the exercises do deal with specific functions). The course does not cover detailed instructions that we are used to on evaluating series and integrals and on special functions. But it is also not very abstract; it truly is mostly complex analysis, not general spaces. The proofs are informed by the more general viewpoint, and there is a strong functional-analysis flavor. For example, much use is made of the Hahn-Banach Theorem and some use of the Urysohn lemma and Tietze extension theorem.

The course was aimed at first-year graduates and has been offered successfully for many first-year graduate students, and we think that is still about the right level for it. Undergraduates interested in the subject matter would be better served, before they tackle this course, by a more traditional complex analysis book such as Bak & Newman's Complex Analysis or Ahlfors's more advanced Complex Analysis, and by one of the many good introductions to Lebesgue integration, e.g. A Primer of Real Functions.

The following Book will be used as a reference for this course. The table of contents is listed on the next pages



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