Course Description of
Ordinary Differential Equations (ODE)

Many interesting and important real life problems are modeled using ordinary differential equations (ODE). These include, but are not limited to, physics, chemistry, biology, engineering, economics, sociology, psychology etc. In mathematics, ODE have a deep connection with geometry, among other branches. In many of these situations, we are interested in understanding the future, given the present phenomenon. In other words, we wish to understand the time evolution or the dynamics of a given phenomenon. The subject field of ODE has developed, over the years, to answer adequately such questions. Yet, there are many important intriguing situations, where complete answers are still awaited. The present book aims at giving a good foundation for a beginner, starting at an undergraduate level, without compromising on the rigour.

We have had several occasions to teach the students at the undergraduate and graduate level in various universities and institutions around the world, including our own institutions, on many topics covered in the course. In our experience and the interactions we have had with the students, we felt that many students lack a clear notion of ODE including the simplest integral calculus problem. For other students, a course on ODE meant learning a few tricks to solve equations. In some countries, in particular, the course which are generally prescribed, consist of a few tricks to solve problems, making ODE one of the most uninteresting subject in the mathematical curriculum. We are of the opinion that many students at the beginning level do not have clarity about the essence of ODE, compared to other subjects in mathematics.

While we were still contemplating to write our updated lecture notes on ODE, to address some of the issues discussed earlier, we got an opportunity to offer an online course on ODE, under the auspices of our own. In this online course, we have presented several topics. We have also tried to address many of the doubts that students may have at the beginning level and the misconceptions some other students may possess.

Is differential equations difficult?

In general, differential equations is considered to be slightly more difficult than calculus 2 (integral calculus). If you did well in calculus 2, it is likely that you can do well in differential equations.

There are actually a number of factors that will impact the difficulty of the class for you. However, there are a number of things that you can do to make it much easier.

Reasons why differential equations can be a hard class

- In differential equations, you will be using equations involving derivates and solving for functions. In calculus 1 you would take the derivative of a function and in calculus 2 you would just integrate the derivative to get the original function.

- As a result, differential equations will involve a lot of integrating and algebra. If you found finding integrals to be difficult in calculus 2, it is more likely that you will have a hard time with differential equations.

- However, there is a lot of material online, now, that you can use to improve your knowledge of integrals and how to do differential equations. So, even if you did have a hard time with calculus 2, it will still be possibly for you to do well with differential equations.

- Also, the equations tend to involve more algebra than most calculus 2 questions. If you struggled with the algebra in calculus 2, we would recommend improving your algebra for differential equations, by just watching Youtube playlists teaching differential equations and making sure to understand how the algebra works as you go. If you can't figure something out, then you could either go back to an algrebra book or just ask on a website like https://math.stackexchange.com/.

- Another reason why differential equations can be difficult is that some professors like to ask questions involving proofs in exams. Compared to a class like real analysis, the proofs in differential equations are not as difficult but they can still be hard.

Even though there are some aspects of differential equations that can be difficult, overall, I would say that differential equations are actually one of the easier higher level math classes.

Reasons why differential equations can be an easy class

- Differential equations tend to be very algorithmic in that you will usually need to identify the type of equation then go through a series of steps for that type of equation to solve it. Once you have practiced on a variety of different types of differential equations, it should become easy for you to identify the type of equation and see the steps you need to take to solve it.

- In addition, differential equations do not tend to be as proof heavy as other higher level math classes. Instead, differential equations are more computational in nature which most students tend to prefer.

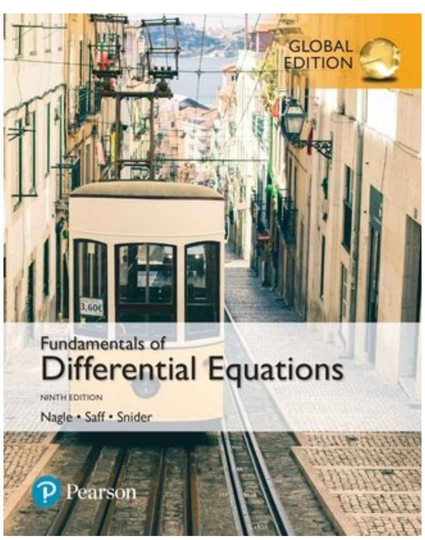
- Another reason why differential equations tend to not be so bad is that there is an abundance of material teaching differential equations online. If you find yourself struggling with a topic in differential equations, it is likely that you will be able to find a lot of resources online that you will be able to use.

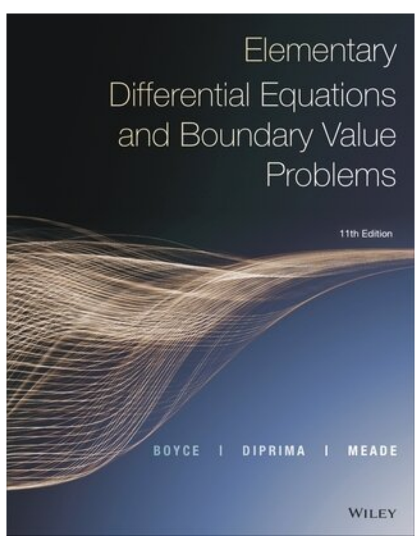
Recommended Textbooks

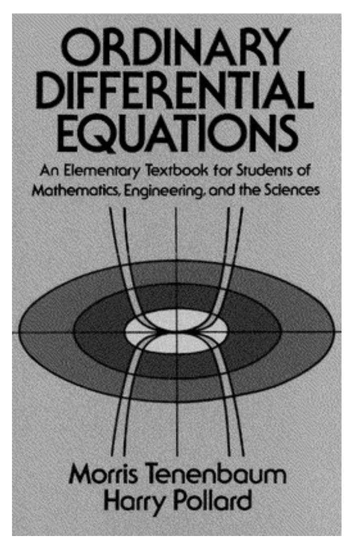
With so many options available, choosing the best textbook for differential equations can be tricky for many students. Students can get overwhelmed by the huge number of options and end in a place by choosing a wrong textbook that will not serve its purpose perfectly.
The purpose of this recommendation is to avoid misunderstanding and help students with a shortlist of the most popular and available textbooks by considering the features, functions, specifications, studentss rating, and reviews.

1. *Ordinary Differential Equations (Dover Books on Mathematics) Revised ed. Edition by Morris Tenenbaum (Author), Harry Pollard (Author), (1985)*
2. *Differential Equations (Schaum's Outlines) 4th Edition, by Richard Bronson (Author), McGraw Hill (2014)*
3. *Introductory Differential Equations: with Boundary Value Problems, 5th Edition, Published By AP (2018)*
4. *Fundamentals of Differential Equations, 9th Edition, (Authors) R. Kent Nagle, Edward B. Saff, Arthur David Snider, Published By Pearson (2019)*
5. *Elementary Differential Equations and Boundary Value Problems, 11th Edition, Author(s) William E. Boyce, Richard C. Diprima, Douglas B. Meade, Published By Wiely (2017)*

NOTE: For this course our tutors will mostly use (4) as a reference for teaching the contents of this course. However, when needed, they will also choose complementary materials from (5) and (1). Prefaces and Table of Contents of these three textbooks will be given in the next pages, respectively.







Preface

Our Goal

Fundamentals of Differential Equations is designed to serve the needs of a one-semester course in basic theory as well as applications of differential equations. The flexibility of the text provides the instructor substantial latitude in designing a syllabus to match the emphasis of the course. Sample syllabi are provided in this preface that illustrate the inherent flexibility of this text to balance theory, methodology, applications, and numerical methods, as well as the incorporation of commercially available computer software for this course.

New to This Edition

- This text now features a MyLab Mathematics course with approximately 750 algorithmic online homework exercises, tutorial videos, and the complete eText. Please see the "Technology and Supplements" section below for more details.
- In the Laplace Transforms chapter (7), the treatments of discontinuous and periodic functions are now divided into two sections that are more appropriate for 50 minute lectures: Section 7.6 "Transforms of Discontinuous Functions" (page 405) and Section 7.7 "Transforms of Periodic and Power Functions" (page 414).
- New examples have been added dealing with variation of parameters, Laplace transforms, the Gamma function, and eigenvectors (among others).
- New problems added to exercise sets deal with such topics as axon gating variables and oscillations of a helium-filled balloon on a cord. Additionally, novel problems accompany the new projects, focusing on economic models, disease control, synchronization, signal propagation, and phase plane analyses of neural responses. We have also added a set of Review Problems for Chapter 1 (page 51).
- Several pedagogical changes were made including amplification of the distinction between phase plane solutions and actual trajectories in Chapter 5 and incorporation of matrix and Jacobian formulations for autonomous systems.
- A new appendix lists commercial software and freeware for direction fields, phase portraits, and numerical methods for solving differential equations. (Appendix G, page 679.)
- "The 2014–2015 Ebola Epidemic" is a new Project in Chapter 5 that describes a system of differential equations for modelling for the spread of the disease in West Africa. The model incorporates such features as contact tracing, number of contacts, likelihood of infection, and efficacy of isolation. See Project F, page 336.
- A new project in Chapter 1 called "Applications to Economics" deals with models for an agrarian economy as well as the growth of capital. See Project C, page 57.

- A new project in Chapter 4 called "Gravity Train" invites to reader to utilize differential equations in the design of an underground tunnel from Moscow to St. Petersburg, Russia, using gravity for propulsion. See Project H, page 262.
- Phase-locked loops constitute the theme of a new project in Chapter 5 that utilizes differential equations to analyze a technique for measuring or matching high frequency radio oscillations. See Project G, page 339.
- A new Project in Chapter 10 broadens the analysis of the wave and heat equations to explore the telegrapher's and cable equations. See Project E, page 659.

Prerequisites

While some universities make linear algebra a prerequisite for differential equations, many schools (especially engineering) only require calculus. With this in mind, we have designed the text so that only Chapter 6 (Theory of Higher-Order Linear Differential Equations) and Chapter 9 (Matrix Methods for Linear Systems) require more than high school level linear algebra. Moreover, Chapter 9 contains review sections on matrices and vectors as well as specific references for the deeper results used from the theory of linear algebra. We have also written Chapter 5 so as to give an introduction to systems of differential equations—including methods of solving, phase plane analysis, applications, numerical procedures, and Poincaré maps—that does not require a background in linear algebra.

Sample Syllabi

As a rough guide in designing a one-semester syllabus related to this text, we provide three samples that can be used for a 15-week course that meets three hours per week. The first emphasizes applications and computations including phase plane analysis; the second is designed for courses that place more emphasis on theory; and the third stresses methodology and partial differential equations. Chapters 1, 2, and 4 provide the core for any first course. The rest of the chapters are, for the most part, independent of each other. For students with a background in linear algebra, the instructor may prefer to replace Chapter 7 (Laplace Transforms) or Chapter 8 (Series Solutions of Differential Equations) with sections from Chapter 9 (Matrix Methods for Linear Systems).

Week	Methods, Computations, and Applications Sections	Theory and Methods (linear algebra prerequisite) Sections	Methods and Partial Differential Equations Sections
1	1.1, 1.2, 1.3	1.1, 1.2, 1.3	1.1, 1.2, 1.3
2	1.4, 2.2	1.4, 2.2, 2.3	1.4, 2.2
3	2.3, 2.4, 3.2	2.4, 3.2, 4.1	2.3, 2.4
4	3.4, 3.5, 3.6	4.2, 4.3, 4.4	3.2, 3.4
5	3.7, 4.1	4.5, 4.6	4.2, 4.3
6	4.2, 4.3, 4.4	4.7, 5.2, 5.3	4.4, 4.5, 4.6
7	4.5, 4.6, 4.7	5.4, 6.1	4.7, 5.1, 5.2
8	4.8, 4.9	6.2, 6.3, 7.2	7.1, 7.2, 7.3
9	4.10, 5.1, 5.2	7.3, 7.4, 7.5	7.4, 7.5
10	5.3, 5.4, 5.5	7.6, 7.7, 7.8	7.6, 7.7
11	5.6, 5.7, 7.2	8.2, 8.3	7.8, 8.2
12	7.3, 7.4, 7.5	8.4, 8.6, 9.1	8.3, 8.5, 8.6
13	7.6, 7.7, 7.8	9.2, 9.3	10.2, 10.3
14	8.1, 8.2, 8.3	9.4, 9.5, 9.6	10.4, 10.5
15	8.4, 8.6	9.7, 9.8	10.6, 10.7

Retained Features

Flexible Organization Most of the material is modular in nature to allow for various course configurations and emphasis (theory, applications and techniques, and concepts).

Optional Use of Computer Software The availability of computer packages such as Mathcad®, Mathematica®, MATLAB®, and Maple™ provides an opportunity for the student to conduct numerical experiments and tackle realistic applications that give additional insights into the subject. Consequently, we have inserted several exercises and projects throughout the text that are designed for the student to employ available software in phase plane analysis, eigenvalue computations, and the numerical solutions of various equations.

Review of Integration In response to the perception that many of today's students' skills in integration have gotten rusty by the time they enter a differential equations course, we have included an appendix offering a quick review of the basic methods for integrating functions analytically.

Choice of Applications Because of syllabus constraints, some courses will have little or no time for sections (such as those in Chapters 3 and 5) that exclusively deal with applications. Therefore, we have made the sections in these chapters independent of each other. To afford the instructor even greater flexibility, we have built in a variety of applications in the exercises for the theoretical sections. In addition, we have included many projects that deal with such applications.

Projects At the end of each chapter are projects relating to the material covered in the chapter. Several of them have been contributed by distinguished researchers. A project might involve a more challenging application, delve deeper into the theory, or introduce more advanced topics in differential equations. Although these projects can be tackled by an individual student, classroom testing has shown that working in groups lends a valuable added dimension to the learning experience. Indeed, it simulates the interactions that take place in the professional arena.

Technical Writing Exercises

Communication skills are, of course, an essential aspect of professional activities. Yet few texts provide opportunities for the reader to develop these skills. Thus, we have added at the end of most chapters a set of clearly marked technical writing exercises that invite students to make documented responses to questions dealing with the concepts in the chapter. In so doing, students are encouraged to make comparisons between various methods and to present examples that support their analysis.

Historical Footnotes

Throughout the text historical footnotes are set off by colored daggers (†). These footnotes typically provide the name of the person who developed the technique, the date, and the context of the original research.

Motivating Problem

Most chapters begin with a discussion of a problem from physics or engineering that motivates the topic presented and illustrates the methodology.

Chapter Summary and Review Problems

All of the main chapters contain a set of review problems along with a synopsis of the major concepts presented.

Computer Graphics

Most of the figures in the text were generated via computer. Computer graphics not only ensure greater accuracy in the illustrations, they demonstrate the use of numerical experimentation in studying the behavior of solutions.

Proofs

While more pragmatic students may balk at proofs, most instructors regard these justifications as an essential ingredient in a textbook on differential equations. As with any text at this level, certain details in the proofs must be omitted. When this occurs, we flag the instance and refer readers either to a problem in the exercises or to another text. For convenience, the end of a proof is marked by the symbol ◆.

Linear Theory

We have developed the theory of linear differential equations in a gradual manner. In Chapter 4 (Linear Second-Order Equations) we first present the basic theory for linear second-order equations with constant coefficients and discuss various techniques for solving these equations. Section 4.7 surveys the extension of these ideas to variable-coefficient second-order equations. A more general and detailed discussion of linear differential equations is given in Chapter 6 (Theory of Higher-Order Linear Differential Equations). For a beginning course emphasizing methods of solution, the presentation in Chapter 4 may be sufficient and Chapter 6 can be skipped.

Numerical Algorithms

Several numerical methods for approximating solutions to differential equations are presented along with program outlines that are easily implemented on a computer. These methods are introduced early in the text so that teachers and/or students can use them for numerical experimentation and for tackling complicated applications. Where appropriate we direct the student to software packages or web-based applets for implementation of these algorithms.

Exercises

An abundance of exercises is graduated in difficulty from straightforward, routine problems to more challenging ones. Deeper theoretical questions, along with applications, usually occur toward the end of the exercise sets. Throughout the text we have included problems and projects that require the use of a calculator or computer. These exercises are denoted by the symbol .

Laplace Transforms

We provide a detailed chapter on Laplace transforms (Chapter 7), since this is a recurring topic for engineers. Our treatment emphasizes discontinuous forcing terms and includes a section on the Dirac delta function.

Power Series Power series solutions is a topic that occasionally causes student anxiety. Possibly, this is due to inadequate preparation in calculus where the more subtle subject of convergent series is (frequently) covered at a rapid pace. Our solution has been to provide a graceful initiation into the theory of power series solutions with an exposition of Taylor polynomial approximants to solutions, deferring the sophisticated issues of convergence to later sections. Unlike many texts, ours provides an extensive section on the method of Frobenius (Section 8.6) as well as a section on finding a second linearly independent solution. While we have given considerable space to power series solutions, we have also taken great care to accommodate the instructor who only wishes to give a basic introduction to the topic. An introduction to solving differential equations using power series and the method of Frobenius can be accomplished by covering the materials in Sections 8.1, 8.2, 8.3, and 8.6.

Partial Differential Equations An introduction to this subject is provided in Chapter 10, which covers the method of separation of variables, Fourier series, the heat equation, the wave equation, and Laplace's equation. Examples in two and three dimensions are included.

Phase Plane Chapter 5 describes how qualitative information for two-dimensional systems can be gleaned about the solutions to intractable autonomous equations by observing their direction fields and critical points on the phase plane. With the assistance of suitable software, this approach provides a refreshing, almost recreational alternative to the traditional analytic methodology as we discuss applications in nonlinear mechanics, ecosystems, and epidemiology.

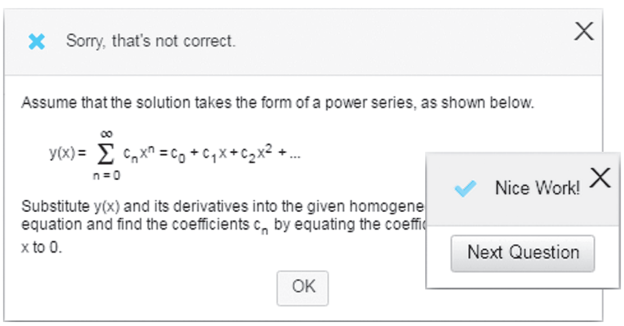
Vibrations Motivation for Chapter 4 on linear differential equations is provided in an introductory section describing the mass–spring oscillator. We exploit the reader's familiarity with common vibratory motions to anticipate the exposition of the theoretical and analytical aspects of linear equations. Not only does this model provide an anchor for the discourse on constant-coefficient equations, but a liberal interpretation of its features enables us to predict the qualitative behavior of variable-coefficient and nonlinear equations as well.

Review of Algebraic Equations and Matrices The chapter on matrix methods for linear systems (Chapter 9) begins with two (optional) introductory sections reviewing the theory of linear algebraic systems and matrix algebra.

Technology and Supplements

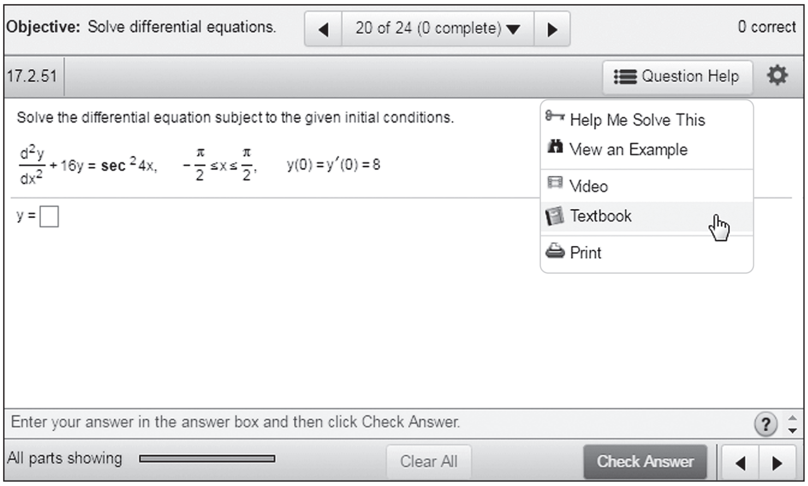
MyLab Mathematics® Online Course (access code required) Built around Pearson's best-selling content, MyLab Mathematics is an online homework, tutorial, and assessment program designed to work with this text to engage students and improve results. MyLab Mathematics can be successfully implemented in any classroom environment—lab-based, hybrid, fully online, or traditional.

MyLab Mathematics's online homework offers students immediate feedback and tutorial assistance that motivates them to do more, which means they retain more knowledge and improve their test scores. Used by more than 37 million students worldwide, MyLab Mathematics delivers consistent, measurable gains in student learning outcomes, retention, and subsequent course success. Visit **www.mymathlab.com/results** to learn more.

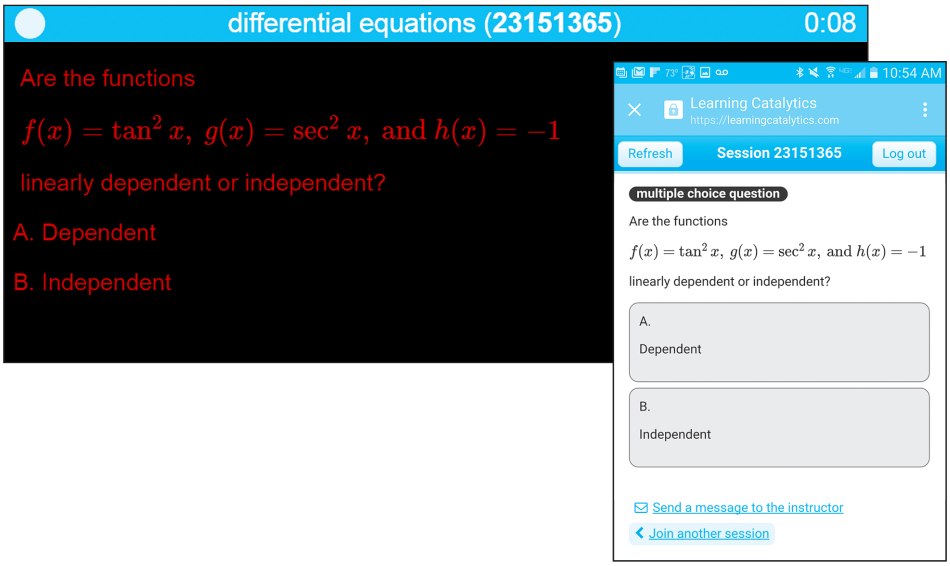


Learning and Teaching Tools

- **Exercises with immediate feedback**—Nearly 750 assignable exercises are based on the textbook exercises and regenerate algorithmically to give students unlimited opportunity for practice and mastery. MyLab Mathematics provides helpful feedback when students enter incorrect answers and includes optional learning aids including Help Me Solve This, View an Example, videos, and an eText.



- **Learning Catalytics™** is a student response tool that uses students' smartphones, tablets, or laptops to engage them in more interactive tasks and thinking. Learning Catalytics fosters student engagement and peer-to-peer learning with real-time analytics.



- **Instructional videos** are available as learning aids within exercises and for self-study within the Multimedia Library. The Guide to Video-Based Assignments makes it easy to assign videos for homework by showing which MyLab Mathematics exercises correspond to each video.
- **The complete eText** is available to students through their MyLab Mathematics courses for the lifetime of the edition, giving students unlimited access to the eText within any course using that edition of the textbook.

- **Accessibility** and achievement go hand in hand. MyLab Mathematics is compatible with the JAWS screen reader, and enables multiple-choice and free-response problem types to be read and interacted with via keyboard controls and math notation input. MyLab Mathematics also works with screen enlargers, including ZoomText, MAGic, and SuperNova. And, all MyLab Mathematics videos have closed-captioning. More information is available at **mymathlab.com/accessibility**.
- **A comprehensive gradebook** with enhanced reporting functionality allows you to efficiently manage your course.
  - **The Reporting Dashboard** provides insight to view, analyze, and report learning outcomes. Student performance data is presented at the class, section, and program levels in an accessible, visual manner so you'll have the information you need to keep your students on track.

  - **Item Analysis** tracks class-wide understanding of particular exercises so you can refine your class lectures or adjust the course/department syllabus. Just-in-time teaching has never been easier!

MyLab Mathematics comes from an experienced partner with educational expertise and an eye on the future. Whether you are just getting started with MyLab Mathematics, or have a question along the way, we're here to help you learn about our technologies and how to incorporate them into your course. To learn more about how MyLab Mathematics helps students succeed, visit **mymathlab.com** or contact your Pearson rep.

Instructor's Solutions Manual (downloadable)

Contains answers to all even-numbered exercises, detailed solutions to the even-numbered problems in several of the main chapters, and additional projects. Available for download in the Pearson Instructor Resource Center as well as within MyLab Mathematics.

MATLAB, Maple, and Mathematica Manuals (downloadable)

By Thomas W. Polaski (Winthrop University), Bruno Welfert (Arizona State University), and Maurino Bautista (Rochester Institute of Technology), respectively. These manuals contain a collection of instructor tips, worksheets, and projects to aid instructors in integrating computer algebra systems into their courses. Complete manuals are available for instructor download within MyLab Mathematics. Student worksheets and projects available for download within MyLab Mathematics.

Contents

CHAPTER 8 Series Solutions of Differential Equations

CHAPTER 9 Matrix Methods for Linear Systems

Preface

As we have prepared an updated edition our first priorities are to preserve, and to enhance, the qualities that have made previous editions so successful. In particular, we adopt the viewpoint of an applied mathematician with diverse interests in differential equations, ranging from quite theoretical to intensely practical – and usually a combination of both. Three pillars of our presentation of the material are methods of solution, analysis of solutions, and approximations of solutions. Regardless of the specific viewpoint adopted, we have sought to ensure the exposition is simultaneously correct and complete, but not needlessly abstract.

The intended audience is undergraduate STEM students whose degree program includes an introductory course in differential equations during the first two years. The essential prerequisite is a working knowledge of calculus, typically a two- or three-semester course sequence or an equivalent. While a basic familiarity with matrices is helpful, Sections 7.2 and 7.3 provide an overview of the essential linear algebra ideas needed for the parts of the book that deal with systems of differential equations (the remainder of Chapter 7, Section 8.5, and Chapter 9).

A strength of this book is its appropriateness in a wide variety of instructional settings. In particular, it allows instructors flexibility in the selection of and the ordering of topics and in the use of technology. The essential core material is Chapter 1, Sections 2.1 through 2.5, and Sections 3.1 through 3.5. After completing these sections, the selection of additional topics, and the order and depth of coverage are generally at the discretion of the instructor. Chapters 4 through 11 are essentially independent of each other, except that Chapter 7 should precede Chapter 9, and Chapter 10 should precede Chapter 11.

A particularly appealing aspect of differential equations is that even the simplest differential equations have a direct correspondence to realistic physical phenomena: exponential growth and decay, spring-mass systems, electrical circuits, competitive species, and wave propagation. More complex natural processes can often be understood by combining and building upon simpler and more basic models. A thorough knowledge of these basic models, the differential equations that describe them, and their solutions – either explicit solutions or qualitative properties of the solution – is the first and indispensable step toward analyzing the solutions of more complex and realistic problems. The modeling process is detailed in Chapter 1 and Section 2.3. Careful constructions of models appear also in Sections 2.5, 3.7, 9.4, 10.5, and 10.7 (and the appendices to Chapter 10). Various problem sets throughout the book include problems that involve modeling to formulate an appropriate differential equation, and then to solve it or to determine some qualitative properties of its solution. The primary purposes of these applied problems are to provide students with hands-on experience in the derivation of differential equations, and to convince them that differential equations arise naturally in a wide variety of real-world applications.

Another important concept emphasized repeatedly throughout the book is the transportability of mathematical knowledge. While a specific solution method applies to only a particular class of differential equations, it can be used in any application in which that particular type of differential equation arises. Once this point is made in a convincing manner, we believe that it is unnecessary to provide specific applications of every method of solution or type of equation that we consider. This decision helps to keep this book to a reasonable size, and allows us to keep the primary emphasis on the development of more solution methods for additional types of differential equations.

From a student's point of view, the problems that are assigned as homework and that appear on examinations define the course. We believe that the most outstanding feature of this book is the number, and above all the variety and range, of the problems that it contains. Many problems are entirely straightforward, but many others are more challenging, and some are fairly open-ended and can even serve as the basis for independent student projects. The observant reader will notice that there are fewer problems in this edition than in previous editions; many of these problems remain available to instructors via the WileyPlus course. The remaining 1600 problems are still far more problems than any instructor can use in any given course, and this provides instructors with a multitude of choices in tailoring their course to meet their own goals and the needs of their students. The answers to almost all of these problems can be found in the pages at the back of the book; full solutions are in either the Student's Solution Manual or the Instructor's Solution Manual.

While we make numerous references to the use of technology, we do so without limiting instructor freedom to use as much, or as little, technology as they desire. Appropriate technologies include advanced graphing calculators (TI Nspire), a spreadsheet (Excel), web-based resources (applets), computer algebra systems, (Maple, Mathematica, Sage), scientific computation systems (MATLAB), or traditional programming (FORTRAN, Javascript, Python). Problems marked with a G are ones we believe are best approached with a graphical tool; those marked with a N are best solved with the use of a numerical tool. Instructors should consider setting their own policies, consistent with their interests and intents about student use of technology when completing assigned problems.

Many problems in this book are best solved through a combination of analytic, graphic, and numeric methods. Pencil-and-paper methods are used to develop a model that is best solved (or analyzed) using a symbolic or graphic tool. The quantitative results and graphs, frequently produced using computer-based resources, serve to illustrate and to clarify conclusions that might not be readily apparent from a complicated explicit solution formula. Conversely, the implementation of an efficient numerical method to obtain an approximate solution typically requires a good deal of preliminary analysis – to determine qualitative features of the solution as a guide to computation, to investigate limiting or special cases, or to discover ranges of the variables or parameters that require an appropriate combination of both analytic and numeric computation. Good judgment may well be required to determine the best choice of solution methods in each particular case. Within this context we point out that problems that request a "sketch" are generally intended to be completed without the use of any technology (except your writing device).

We believe that it is important for students to understand that (except perhaps in courses on differential equations) the goal of solving a differential equation is seldom simply to obtain the solution. Rather, we seek the solution in order to obtain insight into the behavior of the process that the equation purports to model. In other words, the solution is not an end in itself. Thus, we have included in the text a great many problems, as well as some examples, that call for conclusions to be drawn about the solution. Sometimes this takes the form of finding the value of the independent variable at which the solution has a certain property, or determining the long-term behavior of the solution. Other problems ask for the effect of variations in a parameter, or for the determination of all values of a parameter at which the solution experiences a substantial change. Such problems are typical of those that arise in the applications of differential equations, and, depending on the goals of the course, an instructor has the option of assigning as few or as many of these problems as desired.

Readers familiar with the preceding edition will observe that the general structure of the book is unchanged. The minor revisions that we have made in this edition are in many cases the result of suggestions from users of earlier editions. The goals are to improve the clarity and readability of our presentation of basic material about differential equations and their applications. More specifically, the most important revisions include the following:

1. Chapter 1 has been rewritten. Instead of a separate section on the History of Differential Equations, this material appears in three installments in the remaining three section.
2. Additional words of explanation and/or more explicit details in the steps in a derivation have been added throughout each chapter. These are too numerous and widespread to mention individually, but collectively they should help to make the book more readable for many students.
3. There are about forty new or revised problems scattered throughout the book. The total number of problems has been reduced by about 400 problems, which are still available through WileyPlus, leaving about 1600 problems in print.
4. There are new examples in Sections 2.1, 3.8, and 7.5.
5. The majority (is this correct?) of the figures have been redrawn, mainly by the use full color to allow for easier identification of critical properties of the solution. In addition, numerous captions have been expanded to clarify the purpose of the figure without requiring a search of the surrounding text.
6. There are several new references, and some others have been updated.

The authors have found differential equations to be a never-ending source of interesting, and sometimes surprising, results and phenomena. We hope that users of this book, both students and instructors, will share our enthusiasm for the subject.

William E. Boyce and Douglas B. Meade
Watervliet, New York and Columbia, SC
29 August 2016

Supplemental Resources for Instructors and Students

An Instructor's Solutions Manual, ISBN 978-1-119-16976-5, includes solutions for all problems not contained in the Student Solutions Manual.

A Student Solutions Manual, ISBN 978-1-119-16975-8, includes solutions for selected problems in the text.

A Book Companion Site, www.wiley.com/college/boyce, provides a wealth of resources for students and instructors, including

- PowerPoint slides of important definitions, examples, and theorems from the book, as well as graphics for presentation in lectures or for study and note taking.
- Chapter Review Sheets, which enable students to test their knowledge of key concepts. For further review, diagnostic feedback is provided that refers to pertinent sections in the text.
- Mathematica, Maple, and MATLAB data files for selected problems in the text providing opportunities for further exploration of important concepts.
- Projects that deal with extended problems normally not included among traditional topics in differential equations, many involving applications from a variety of disciplines. These vary in length and complexity, and they can be assigned as individual homework or as group assignments.

A series of supplemental guidebooks, also published by John Wiley & Sons, can be used with Boyce/DiPrima/Meade in order to incorporate computing technologies into the course. These books emphasize numerical methods and graphical analysis, showing how these methods enable us to interpret solutions of ordinary differential equations (ODEs) in the real world. Separate guidebooks cover each of the three major mathematical software formats, but the ODE subject matter is the same in each.

- Hunt, Lipsman, Osborn, and Rosenberg, *Differential Equations with MATLAB*, 3rd ed., 2012, ISBN 978-1-118-37680-5

- Hunt, Lardy, Lipsman, Osborn, and Rosenberg, *Differential Equations with Maple*, 3rd ed., 2008, ISBN 978-0-471-77317-7
- Hunt, Outing, Lipsman, Osborn, and Rosenberg, *Differential Equations with Mathematica*, 3rd ed., 2009, ISBN 978-0-471-77316-0

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WILLIAM E. BOYCE AND DOUGLAS B. MEADE

Brief Contents

Contents

Preface for the Teacher

IN WRITING THIS BOOK, it has been our aim to make it readable for the student, to include topics of increasing importance (such as transforms, numerical analysis, the perturbation concept) and to avoid the errors traditionally transmitted in an elementary text. In this last connection, we have abandoned the use of the terminology "general solution" of a differential equation unless the solution is in fact general, i.e., unless the solution actually contains every solution of the differential equation. We have also avoided the term "singular solution." We have exercised great care in defining function, differentials and solutions; in particular we have tried to make it clear that functions have domains.

On the other hand, this accuracy has been secondary to our main purpose: to teach the student how to use differential equations. We hope and believe that we have not overlooked any of the major applications which can be made comprehensible at this elementary level. You will find in this text an extensive list of worked examples and homework problems with answers.

We acknowledge our indebtedness to the publishers for their cooperation and willingness to let us use new pedagogical devices and to Prof. C. A. Hutchinson for his thorough editing.

M.T.
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Ithaca, New York
West Lafayette, Indiana

Preface for the Student

THIS BOOK HAS BEEN WRITTEN primarily for you, the student. We have tried to make it easy to read and easy to follow.

We do not wish to imply, however, that you will be able to read this text as if it were a novel. If you wish to derive any benefit from it, you must study each page slowly and carefully. You must have pencil and plenty of paper beside you so that you yourself can reproduce each step and equation in an argument. When we say "verify a statement," "make a substitution," "add two equations," "multiply two factors," etc., you yourself must actually perform these operations. If you carry out the explicit and detailed instructions we have given you, we can almost guarantee that you will, with relative ease, reach the conclusion.

One final suggestion—as you come across formulas, record them and their equation numbers on a separate sheet of paper for easy reference. You may also find it advantageous to do the same for Definitions and Theorems.

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Contents