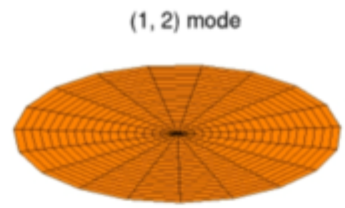
Course Description of Functional Analysis

What is
Functional analysis

Functional analysis is a branch of mathematical analysis, the core of which is formed by the study of vector spaces endowed with some kind of limit-related structure (for example, inner product, norm, or topology) and the linear functions defined on these spaces and suitably respecting these structures. The historical roots of functional analysis lie in the study of spaces of functions and the formulation of properties of transformations of functions such as the Fourier transform as transformations defining, for example, continuous or unitary operators between function spaces. This point of view turned out to be particularly useful for the study of differential and integral equations.



One of the possible modes of vibration of an idealized circular drum head. These modes are eigenfunctions of a linear operator on a function space, a common construction in functional analysis.

The usage of the word *functional* as a noun goes back to the calculus of variations, implying a function whose argument is a function. The term was first used in Hadamard's 1910 book on that subject. However, the general concept of a functional had previously been introduced in 1887 by the Italian mathematician and physicist Vito Volterra.[1][2] The theory of nonlinear functionals was continued by students of Hadamard, in particular Fréchet and Lévy. Hadamard also founded the modern school of linear functional analysis further developed by Riesz and the group of Polish mathematicians around Stefan Banach.

In modern introductory texts on functional analysis, the subject is seen as the study of vector spaces endowed with a topology, in particular infinite-dimensional spaces.[3][4] In contrast, linear algebra deals mostly with finite-dimensional spaces, and does not use topology. An important part of functional analysis is the extension of the theories of measure, integration, and probability to infinite dimensional spaces, also known as **infinite dimensional analysis**.

Course Description:

The unit aims to provide students with a firm grounding in the theory and techniques of Functional Analysis and to offer students ample opportunity to build on their problem-solving ability in this subject. It also aims to equip students with independent self-study and presentation-giving skills. This course sets out to explore some core notions in Functional Analysis which originated in the study of integral/differential equations and more generally equations for operators in infinite dimensional spaces. These techniques can be helpful, for instance, in analysing trigonometric series and can be used to make sense of the determinant of an infinite-dimensional matrix. Functional Analysis has found broad applicability in diverse areas of mathematics, physics, economics, and other sciences. Students will be introduced to the theory of Banach and Hilbert spaces. The highlight of the course will be an exposition of the four fundamental theorems in the Functional Analysis (Hahn-Banach theorem, uniform boundedness theorem, open mapping theorem, closed graph theorem). The unit may also include some discussion of the spectral theory of linear operators.

Text Book
Functional Analysis 1991
Walter Rudin

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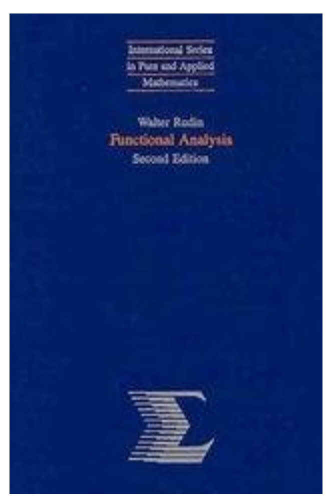

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