

Course Description of Functional Analysis

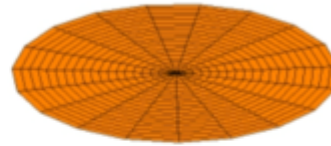
What is Functional analysis

Functional analysis is a branch of mathematical analysis, the core of which is formed by the study of vector spaces endowed with some kind of limit-related structure (for example, inner product, norm, or topology) and the linear functions defined on these spaces and suitably respecting these structures. The historical roots of functional analysis lie in the study of spaces of functions and the formulation of properties of transformations of functions such as the Fourier transform as transformations defining, for example, continuous or unitary operators between function spaces. This point of view turned out to be particularly useful for the study of differential and integral equations.

The usage of the word *functional* as a noun goes back to the calculus of variations, implying a function whose argument is a function. The term was first used in Hadamard's 1910 book on that subject. However, the general concept of a functional had previously been introduced in 1887 by the Italian mathematician and physicist Vito Volterra.^{[1][2]} The theory of nonlinear functionals was continued by students of Hadamard, in particular Fréchet and Lévy. Hadamard also founded the modern school of linear functional analysis further developed by Riesz and the group of Polish mathematicians around Stefan Banach.

In modern introductory texts on functional analysis, the subject is seen as the study of vector spaces endowed with a topology, in particular infinite-dimensional spaces.^{[3][4]} In contrast, linear algebra deals mostly with finite-dimensional spaces, and does not use topology. An important part of functional analysis is the extension of the theories of measure, integration, and probability to infinite dimensional spaces, also known as **infinite dimensional analysis**.

(1, 2) mode



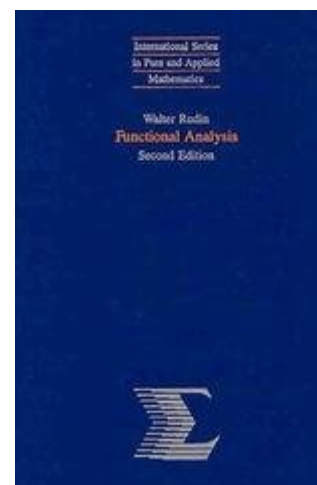
One of the possible modes of vibration of an idealized circular drum head. These modes are eigenfunctions of a linear operator on a function space, a common construction in functional analysis.

Course Description:

The unit aims to provide students with a firm grounding in the theory and techniques of Functional Analysis and to offer students ample opportunity to build on their problem-solving ability in this subject. It also aims to equip students with independent self-study and presentation-giving skills. This course sets out to explore some core notions in Functional Analysis which originated in the study of integral/differential equations and more generally equations for operators in infinite dimensional spaces. These techniques can be helpful, for instance, in analysing trigonometric series and can be used to make sense of the determinant of an infinite-dimensional matrix. Functional Analysis has found broad applicability in diverse areas of mathematics, physics, economics, and other sciences. Students will be introduced to the theory of Banach and Hilbert spaces. The highlight of the course will be an exposition of the four fundamental theorems in the Functional Analysis (Hahn-Banach theorem, uniform boundedness theorem, open mapping theorem, closed graph theorem). The unit may also include some discussion of the spectral theory of linear operators.

Text Book Functional Analysis 1991 Walter Rudin

Table of contents is listed in the following pages



Part I General Theory

1	Topological Vector Spaces	3
	Introduction	3
	Separation properties	10
	Linear mappings	14
	Finite-dimensional spaces	16
	Metrization	18
	Boundedness and continuity	23
	Seminorms and local convexity	25
	Quotient spaces	30
	Examples	33
	Exercises	38
2	Completeness	42
	Baire category	42
	The Banach-Steinhaus theorem	43
	The open mapping theorem	47
	The closed graph theorem	50
	Bilinear mappings	52
	Exercises	53
3	Convexity	56
	The Hahn-Banach theorems	56
	Weak topologies	62
	Compact convex sets	68
	Vector-valued integration	77
	Holomorphic functions	82
	Exercises	85

4	Duality in Banach Spaces	92
	The normed dual of a normed space	92
	Adjoint	97
	Compact operators	103
	Exercises	111
5	Some Applications	116
	A continuity theorem	116
	Closed subspaces of L^p -spaces	117
	The range of a vector-valued measure	120
	A generalized Stone-Weierstrass theorem	121
	Two interpolation theorems	124
	Kakutani's fixed point theorem	126
	Haar measure on compact groups	128
	Uncomplemented subspaces	132
	Sums of Poisson kernels	138
	Two more fixed point theorems	139
	Exercises	144

Part II Distributions and Fourier Transforms

6	Test Functions and Distributions	149
	Introduction	149
	Test function spaces	151
	Calculus with distributions	157
	Localization	162
	Supports of distributions	164
	Distributions as derivatives	167
	Convolutions	170
	Exercises	177
7	Fourier Transforms	182
	Basic properties	182
	Tempered distributions	189
	Paley-Wiener theorems	196
	Sobolev's lemma	202
	Exercises	204
8	Applications to Differential Equations	210
	Fundamental solutions	210
	Elliptic equations	215
	Exercises	222