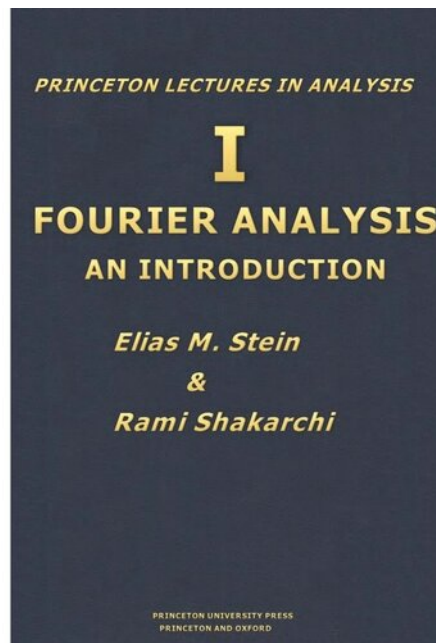


Course Description of Fourier Analysis and its Application

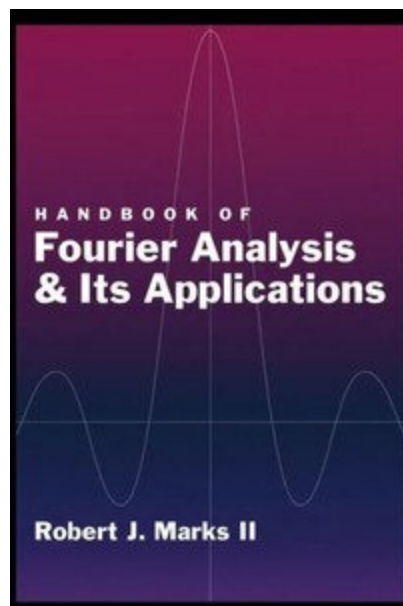
Fourier Analysis is a powerful tool for many problems, and especially for solving various differential equations of interest in science and engineering. The techniques presented in this course are well suited to study problems such as the vibrations of a stretched string (e.g. guitarstring), the vibrations of a stretched membrane (e.g. drumhead), the waves in an incompressible fluid, electromagnetic waves (e.g. light or radio waves); the diffusion of heat or the minimization of certain energies. This is a course in *applicable mathematics*.

The study of Fourier analysis can serve as a capstone course in undergraduate mathematics, because it builds on such a variety of mathematical topics. Solving ordinary differential equations is the most crucial prerequisite, but ideas from many other courses are useful. Some aspects of Fourier series are best understood by regarding the coefficients in the series as components of an infinite dimensional vector. Then linear algebra becomes useful, in particular ideas about eigenvalues and eigenvectors. Convergence properties for sequences and series enter the course as described above, and vector calculus occasionally makes an appearance.

The contents that will be covered in this course are from the following book and are listed in the next pages.



To show further applications of Fourier Analysis, we will choose appropriate examples from the following book.



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