Description of Elementary Functional Analysis Course

Functional analysis helps us study and solve both linear and nonlinear problems posed on a normed space that is no longer finite-dimensional, a situation that arises very naturally in many concrete problems. For example, a nonrelativistic quantum particle confined to a region in space can be modeled using a complex valued function (a wave function), an infinite dimensional object (the function's value is required for each of the infinitely many points in the region). Functional analysis yields the mathematically and physically interesting fact that the (time independent) state of the particle can always be described as a (possibly infinite) superposition of elementary wave functions (bound states) that form a discrete set and can be ordered to have increasing energies tending to infinity.

The fundamental topics from functional analysis covered in this course include normed spaces, completeness, functionals, the Hahn-Banach Theorem, duality, operators; Lebesgue measure, measurable functions, integrability, completeness of L^p spaces; Hilbert spaces; compact and self-adjoint operators; and the Spectral Theorem.

The following contents will be covered in this course

- Basic Banach Space Theory
- Bounded Linear Operators
- Quotient Spaces, the Baire Category Theorem and the Uniform Boundedness Theorem
- The Open Mapping Theorem and the Closed Graph Theorem
- Zorn's Lemma and the Hahn-Banach Theorem
- The Double Dual and the Outer Measure of a Subset of Real Numbers
- Sigma Algebras
- Lebesgue Measurable Subsets and Measure
- Lebesgue Measurable Functions
- Simple Functions
- The Lebesgue Integral of a Nonnegative Function and Convergence Theorems
- Lebesgue Integrable Functions, the Lebesgue Integral and the Dominated Convergence Theorem
- L^p Space Theory
- Basic Hilbert Space Theory
- Orthonormal Bases and Fourier Series
- Fejer's Theorem and Convergence of Fourier Series
- Minimizers, Orthogonal Complements and the Riesz Representation Theorem
- The Adjoint of a Bounded Linear Operator on a Hilbert Space
- Compact Subsets of a Hilbert Space and Finite-Rank Operators
- Compact Operators and the Spectrum of a Bounded Linear Operator on a Hilbert Space
- The Spectrum of Self-Adjoint Operators and the Eigenspaces of Compact Self-Adjoint Operators
- The Spectral Theorem for a Compact Self-Adjoint Operator
- The Dirichlet Problem on an Interval

We use the following book to instruct and illustrate all of the above contents.